

**Exam III**  
**MATH 155 Section 08**  
**December 17th, 2015. 6:20PM–8:20PM**

**Your name:**

**Instructions:** Please clearly write your name above. This exam is closed-book and closed-note. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Write all details explicitly. Answers without justifications and/or calculation steps may receive no score. Hand-in this exam sheets and other sheets which contain your work to be graded. Cross out everything which you do not want them to be graded.

Total 100 points. 10 points each unless specified otherwise.

1. (5 points) Let  $R$  be the region in the  $xy$ -plane bounded by the curves  $y = \sqrt{x}$  and  $y = x^2$ . Set up an integral which equals the volume of the solid formed by rotating the region  $R$  around the  $x$ -axis. Do not evaluate the integral.

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2. (10 points) Prove by using integration that the surface area of the sphere with radius  $R$  is  $4\pi R^2$ .

3. (10 points) Calculate the following integral:

$$\int e^{3x} \cos 2x dx.$$

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4. (5 points each) Evaluate or show divergence:

(1)

$$\int_1^{\infty} \frac{1}{\sqrt{x}} dx$$

(2)

$$\int_0^{\infty} e^{-x} dx$$

5. (5 points each) Compute the limit of the sequence or show divergence:

(1)

$$\lim_{k \rightarrow \infty} \frac{e^k}{k^2}.$$

(2)

$$\lim_{n \rightarrow \infty} \frac{\cos n}{n}.$$

(3)

$$\lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{3}{2^k}.$$

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6. Given an infinite series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 1},$$

show that the series is convergent using indicated methods:

(1) (3 points) The comparison test. (You can use  $\sum_{n=1}^{\infty} \frac{1}{n^2}$  is convergent without proof.)

(2) (7 points) The integral test. (You should compute a finite integral you need for comparison.)

7. (5 points) Show that the alternating Harmonic series  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$  is convergent.

8. (10 points) Write down the degree 4 Taylor polynomial centered at 0:

$$p_4(x) = \sum_{k=0}^4 \frac{f^{(k)}(0)}{k!} x^k$$

for given  $f(x) = 1 + \cos x$ .

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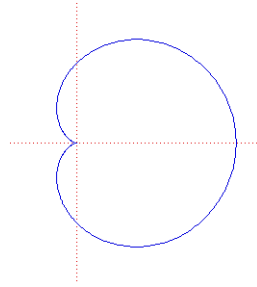
9. (10 points) Find the interval of convergence of the power series:

$$\sum_{n=2}^{\infty} \frac{5(x-2)^n}{n-1}.$$

(Clearly mention whether your final answer is a(n) open, half-open, or closed interval!)

10. (1) (5 points) Let  $C$  be a circle of radius 2 centered at  $(2, 0)$ . Write the equation of  $C$  in the polar coordinate.





(2) (10 points) Calculate the enclosed area by the cardioid  $r = 1 + \cos \theta$  depicted as above.

Please use this page if you need more space.

**MATH155-08 Exit Survey:** This set of questions does not mean to pry on your privacy. Feel free to leave it blank if you feel uncomfortable about answering it. Answers in this section will not affect at all on any kind of evaluation in this course. It will be used confidentially for the instructor to accommodate students better when he teaches a late afternoon or a night class next time.

(1) I am going to take Calculus with Analytic Geometry 3 eventually at Hunter College.

Yes,  No,  Not sure

(2) In addition to holding a student status, I work.

Yes,  No

(3) If yes, I work:

full-time  20 to 39 hours a week  10 to 19 hours a week  0 to 9 hours a week.

(4) How many credits do you take in this semester?

More than 15  10 to 15 credits  5 to 9 credits  I take only this course.

PLEASE DETACH AND SUBMIT SEPARATELY. **DO NOT** WRITE YOUR NAME ON THIS PAGE. THANK YOU!