

Review Problems for Exam II (Revised version)
MATH 155 Section 06
Exam Date and Time: April 4th, 2017. 09:00–11:00

REVIEW PROBLEMS

1. Calculate the following integral:

$$\int \sec^3 \theta d\theta$$

Hint: You may use $\int \sec \theta d\theta = \ln |\sec \theta + \tan \theta| + C$.

2. Calculate the following integral:

$$\int \sin^4 x dx$$

3. Prove that the area of an ellipse whose equation is given by $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $ab\pi$.

4. Calculate

$$\int \frac{x^2}{\sqrt{4+x^2}} dx.$$

5. Calculate

$$\int \frac{dx}{x^2 - x - 2}.$$

6. Find the constant k that satisfies the following equation:

$$\int_{-\infty}^{\infty} \frac{k}{4+9x^2} dx = 1.$$

7. (5 points each) Let $f(x) = \frac{1}{x^p}$, where $0 < p < \infty$. Discuss the convergence of the definite integral $\int_1^{\infty} f(x) dx$ in the following cases:

(1) When $0 < p < 1$, (2) When $p = 1$, and (3) When $p > 1$.

8. Find the value that the following infinite sum converges to:

$$\sum_{n=1}^{\infty} \frac{2}{4n^2 - 1}.$$

9. Find the limit of the sequence as $n \rightarrow \infty$:

$$a_n = \frac{\sin n}{n}.$$

- 10 (5 points). Evaluate the following geometric series: $1 + \frac{2}{7} + \frac{2^2}{7^2} + \dots + \frac{2^n}{7^n} + \dots$