Midterm Exam

MATH 25500 Section 01

24th March 2017, 14:10–15:25

Your name:

Instructions: Please clearly write your name above. You may use your textbook or notes. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Please write all details explicitly. Answers without justifications and/or calculation steps may receive no score. 10 points each unless stated otherwise.

1. Let $f, g : \mathbb{R}^3 \to \mathbb{R}$ be C^1-functions. Show that $\nabla(fg) = f\nabla g + g\nabla f$.

In the #2 and #3 of this exam, $\overrightarrow{F}(x,y,z) = (x,y,z)$ is a vector field on $\mathbb{R}^3$.

2. Find $\nabla \cdot \overrightarrow{F}$ and $\nabla \times \overrightarrow{F}$.

3. Let $\alpha(t) = (t\sin t, t\cos t, t)$ be a curve in $\mathbb{R}^3$ defined on $[0, 2\pi]$. Calculate $\int_\alpha \overrightarrow{F} \cdot d\overrightarrow{r}$.

From here to the end of this exam, $\Phi : D \subset \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$(u, v) \mapsto (v \sin u, v \cos u, v)$$

is a parametrization of a surface S, where $D = [0, 2\pi] \times [0, 1]$.

4. Find all points $(u, v) \in D$ such that the surface S is regular at $\Phi(u, v)$.

5. Find the area of S.

6. Evaluate $\iint_S x dS$.

7. Compute the surface integral $\iint_S \overrightarrow{G} \cdot d\overrightarrow{S}$, where $\overrightarrow{G}(x, y, z) = (x, y, z^2)$.

8. (20 points) Calculate the Gauss curvature and the mean curvature at each point $p \in S - \{(0,0,0)\}$.

9. The temperature distribution in $\mathbb{R}^3$ is given by $T(x, y, z) = 4x^2 + 4y^2$. Compute the heat flux across the surface $x^2 + y^2 = 1$ and $0 \leq z \leq 1$. Here the heat flux $\overrightarrow{F}$ is defined by $\overrightarrow{F} := -\nabla T$.