Homework 11

MATH 25500 Section 01

Due: 12th May 2017

You can look up any textbook on linear algebra or wikipedia pages. A good reference: Serge Lang, *Linear Algebra*, 3rd edition, Springer-Verlag New York.

1. Give a definition of a **vector space** over $\mathbb{R}$. (In other words a $\mathbb{R}$-vector space.)

2. Give an example of a vector space over $\mathbb{R}$.

3. Give a definition of a **scalar product** on a vector space V over $\mathbb{R}$. In other words, give a definition of a symmetric bilinear form on V over $\mathbb{R}$.

4. Give an example of a scalar product on a vector space V over $\mathbb{R}$.

5. Give a definition of a **linear map** from a $\mathbb{R}$-vector space V to a $\mathbb{R}$-vector space W.

6. Consider a set V^* defined by

$$\{\varphi \in V^* | \varphi : V \to \mathbb{R} \text{ is a linear map}\}.$$

Give a canonical $\mathbb{R}$-vector space structure on V^* and verify that, with the vector space structure you have given, V^* is a $\mathbb{R}$-vector space.