In Exercises 1 to 8, sketch the given vector field or a small multiple of it.

7. $F(x,y)=\left(\frac{x}{\sqrt{x^2+y^2}},\frac{y}{\sqrt{x^2+y^2}}\right)$

Our vector field is $f(x,y)=\left(\frac{x}{\sqrt{x^2+y^2}},\frac{y}{\sqrt{x^2+y^2}}\right)$.

We need to sketch the vector field or a small part of it.

A vector field in $\mathbb{R}^n$ is a map

$F: A\subset \mathbb{R}^n \longrightarrow \mathbb{R}^n$ that ~~assigns~~ assigns to each point x in its domain A, a vector $F(x)$.

We can rewrite the given vector field as follows

$$f(x,y)=\frac{x}{\sqrt{x^2+y^2}}\hat{i}+\frac{y}{\sqrt{x^2+y^2}}\hat{j}$$

$$\nabla\cdot F=\left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}\frac{x}{\sqrt{x^2+y^2}}+\hat{j}\frac{y}{\sqrt{x^2+y^2}}\right)$$

$$=\frac{\partial}{\partial x}\frac{x}{\sqrt{x^2+y^2}}+\frac{\partial}{\partial y}\frac{y}{\sqrt{x^2+y^2}}$$

$$\Downarrow$$

$$=\frac{\sqrt{x^2+y^2}\cdot 1-x\frac{1(2x)}{2\sqrt{x^2+y^2}}}{\left(\sqrt{x^2+y^2}\right)^2}+\frac{\sqrt{x^2+y^2}\cdot 1-y\frac{1(2y)}{2\sqrt{x^2+y^2}}}{\left(\sqrt{x^2+y^2}\right)^2}.$$

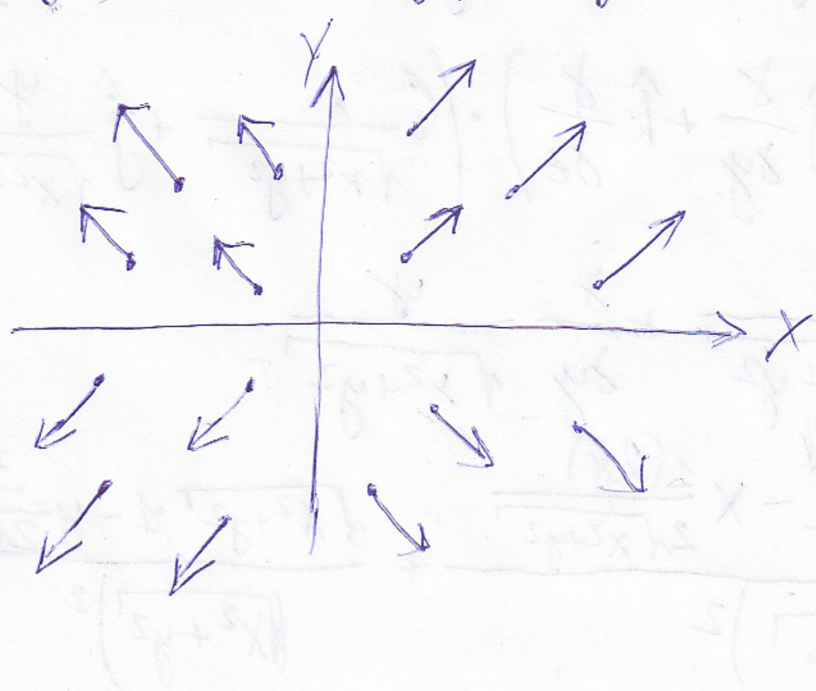
7. CONTINUATION

$$= \frac{\left[\sqrt{x^2+y^2} - \frac{x^2}{\sqrt{x^2+y^2}} + \sqrt{x^2+y^2} - \frac{y^2}{\sqrt{x^2+y^2}}\right]}{(x^2+y^2)}$$

$$= \frac{\left[\frac{(\sqrt{x^2+y^2})^2}{\sqrt{x^2+y^2}} - \frac{x^2}{\sqrt{x^2+y^2}} + \frac{(\sqrt{x^2+y^2})^2}{\sqrt{x^2+y^2}} - \frac{y^2}{\sqrt{x^2+y^2}}\right]}{(x^2+y^2)}$$

$$= \frac{\left[\frac{x^2+y^2-x^2}{\sqrt{x^2+y^2}} + \frac{x^2+y^2-y^2}{\sqrt{x^2+y^2}}\right]}{(x^2+y^2)}$$

$$= \frac{\frac{y^2+x^2}{\sqrt{x^2+y^2}}}{(x^2+y^2)} = \frac{y^2+x^2}{(x^2+y^2)\sqrt{x^2+y^2}} = \frac{\sqrt{x^2+y^2}}{(x^2+y^2)} > 0$$



DARIUSZ SIERGIEJUK 4.3 VECTOR FIELDS HOMEWORK #1

8. $F(x,y) = \left(\frac{y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}}\right)$

We are provided with the above vector field $F(x,y) = \left(\frac{y}{\sqrt{x^2+y^2}}, \frac{x}{\sqrt{x^2+y^2}}\right)$

We need to sketch the vector field or a small part of it.

We can rewrite the given vector field as follows,

$$F(x,y) = \frac{y}{\sqrt{x^2+y^2}}\hat{i} + \frac{y}{\sqrt{x^2+y^2}}\hat{j}$$

The divergence of the vector field is,

$$\nabla \cdot F = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}\right) \cdot \left(\hat{i}\frac{y}{\sqrt{x^2+y^2}} + \hat{j}\frac{x}{\sqrt{x^2+y^2}}\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{y}{\sqrt{x^2+y^2}}\right) + \frac{\partial}{\partial y}\left(\frac{x}{\sqrt{x^2+y^2}}\right)$$

Rewriting

$$= \frac{\partial}{\partial x}\left(y(x^2+y^2)^{-\frac{1}{2}}\right) + \frac{\partial}{\partial y}\left(x(x^2+y^2)^{-\frac{1}{2}}\right)$$

$$= y\left(\left(-\frac{1}{2}\right)(x^2+y^2)^{-\frac{3}{2}}(2x)\right) + x\left(\left(-\frac{1}{2}\right)(x^2+y^2)^{-\frac{3}{2}}(2y)\right)$$

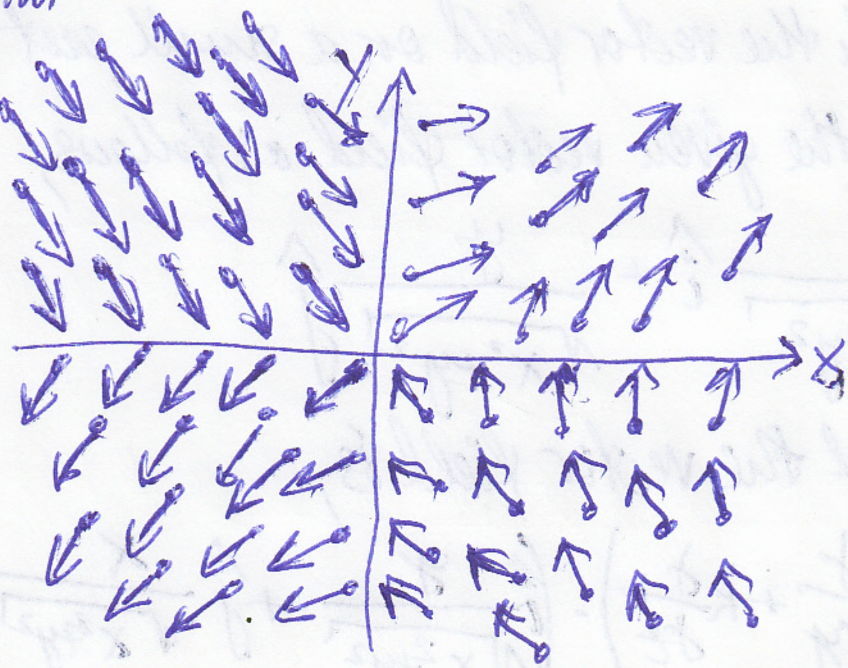
$$= y\left((x^2+y^2)^{-3/2}(-x)\right) + x\left((x^2+y^2)^{-3/2}(-y)\right)$$

$$= \frac{-xy}{(x^2+y^2)^{3/2}} - \frac{xy}{(x^2+y^2)^{3/2}} = -\frac{2xy}{(x^2+y^2)\sqrt{x^2+y^2}} < 0$$

DARIUSZ SIE~~Ś~~IEJUK 4.3 VECTOR FIELDS HOMEWORK #1 ④

8. CONTINUATION

Since the divergence of the vector field is less than zero, the flow of the given vector field, F, is directed towards the origin.



DARIUSZ SIERGIEJUK 4.3 VECTOR FIELDS HOMEWORK #1 (5)

In Exercises 15 to 18, show that the curve $c(t)$ is a flow line of the given velocity vector field $F(x,y,z)$.

16. $c(t) = (t^2, 2t-1, \sqrt{t}), t>0 : F(x,y,z) = (y+1, 2, 1/2z)$

We are given the above curve,

$c(t) = (t^2, 2t-1, \sqrt{t}), t>0$

and vector field $F(x,y,z) = (y+1, 2, \frac{1}{2z})$

We need to show that the provided curve $c(t)$ is a flow line of the velocity vector field $F(x,y,z)$.

$F(c(t)) = c'(t)$, then the given curve is a flow line of the given velocity vector field.

$$c(t) = t^2\hat{i} + (2t-1)\hat{j} + \sqrt{t}\,\hat{k}$$

$$c'(t) = 2t\hat{i} + 2\hat{j} + \frac{1}{2\sqrt{t}}\hat{k}$$

$$F(x,y,z) = (y+1)\hat{i} + 2\hat{j} + \frac{1}{2z}\hat{k}$$

$$F(c(t)) = 2t\hat{i} + 2\hat{j} + \frac{1}{2\sqrt{t}}\hat{k}$$

Hence, $c'(t) = F(c(t))$

This curve $c(t)$ has a flow line.

17. $c(t)=(\sin t, \cos t, e^t)$; $F(x,y,z)=(y,-x,z)$

Our curve and the velocity vector are as provided,

$c(t)=(\sin t, \cos t, e^t)$

$F(x,y,z)=(y,-x,z)$

Our task is to prove that the given curve $c(t)$ is a flow line of the provided velocity vector field $F(x,y,z)$.

If $F(c(t))=c'(t)$, then the particular curve is a flow line of the particular velocity vector field.

$c(t)=\sin t\hat{i}+\cos t\hat{j}+e^t\hat{k}$

$c'(t)=\frac{d}{dt}(\sin t\hat{i}+\cos t\hat{j}+e^t\hat{k})$

$c'(t)=\cos t\hat{i}-\sin t\hat{j}+e^t\hat{k}$

$F(x,y,z)=y\hat{i}-x\hat{j}+z\hat{k}$

$F(c(t))=\cos t\hat{i}-\sin t\hat{j}+e^t\hat{k}$

Here, $c'(t)=F(c(t))$

Therefore the curve $c(t)$ has a flow line.

21. (a) Let $F(x,y,z)=(yz,xz,xy)$.
Find a function $f:\mathbb{R}^3\longmapsto\mathbb{R}$ such that $F=\nabla f$.

Our vector field here $F(x,y,z)=(yz,xz,xy)$.
We need to locate the function $f:\mathbb{R}^3\longmapsto\mathbb{R}$ such that $F=\nabla f$.

The gradient of this function f is,
$$\nabla f=\frac{\partial f}{\partial x}\hat{i}+\frac{\partial f}{\partial y}\hat{j}+\frac{\partial f}{\partial z}\hat{k}.$$

Let assume that $F=\nabla f$, therefore $\frac{\partial f}{\partial x}=yz$, $\frac{\partial f}{\partial y}=xz$, $\frac{\partial f}{\partial z}=xy$.

Integrating $\frac{\partial f}{\partial x}=yz$ with respect to x partially,
$$f(x,y,z)=xyz+\phi_1(y,z)$$
Integrating $\frac{\partial f}{\partial y}=xz$ with respect to y partially,
$$f(x,y,z)=xyz+\phi_2(x,z)$$
Integrating $\frac{\partial f}{\partial z}=xy$ with respect to z partially,
$$f(x,y,z)=xyz+\phi_3(x,y)$$
Choosing $f(x,y,z)=xyz+C$ where C is a constant.
The function is $F=\nabla f$.

21.(a) CONTINUATION

Therefore, a function satisfying the condition $f:\mathbb{R}^3 \mapsto \mathbb{R}$ is $\boxed{f(x,y,z)=xyz+C}$.

24.(b) Let $F(x,y,z)=(x,y,z)$.

Find a function $f:\mathbb{R}^3 \mapsto \mathbb{R}$ such that $F=\nabla f$.

Our vector field here is $F(x,y,z)=(x,y,z)$.

We have to locate a function $f:\mathbb{R}^3 \mapsto \mathbb{R}$ such that $F=\nabla f$.

The gradient of this function f is,

$$\nabla f = \frac{\partial f}{\partial x}\hat{i}+\frac{\partial f}{\partial y}\hat{j}+\frac{\partial f}{\partial z}\hat{k}.$$

Let us assume that $F=\nabla f$, therefore $\frac{\partial f}{\partial x}=x$, $\frac{\partial f}{\partial y}=y$, $\frac{\partial f}{\partial z}=z$.

Integrating $\frac{\partial f}{\partial x}=x$ with respect to x partially,

$$f(x,y,z)=\frac{x^2}{2}+\phi_1(y,z)$$

Integrating $\frac{\partial f}{\partial y}=y$ with respect to y partially,

$$f(x,y,z)=\frac{y^2}{2}+\phi_2(x,z)$$

21.(b) CONTINUATION

Integrating $\frac{\partial f}{\partial z} = z$ with respect to z partially,

$$f(x,y,z) = \frac{z^2}{2} + \phi_3(x,y)$$

Selecting $f(x,y,z) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} + C$ where C is a constant. The function here is $F = \nabla f$.

Hence, this function meets the condition,

$f : \mathbb{R}^3 \to \mathbb{R}$ is $\boxed{f(x,y,z) = \frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} + C}$

24. Let $c(t)$ be a flow line of a gradient field $F = -\nabla V$. Prove that $V(c(t))$ is a decreasing function of t.

Let us assume that $c(t)$ is a flow line of a gradient field $F = -\nabla V$.

We are required to show that $V(c(t))$ is a decreasing function of t.

The flow line of a gradient vector field is a path $c(t)$.

$$c'(t) = F(c(t)).$$

$$= -\nabla \cdot V(c(t))$$

Where $\nabla \cdot V(c(t))$ is the gradient vector field of $V(c(t))$.

In this instance here $-\nabla \cdot V(c(t))$ denotes the decreasing direction of $V(c(t))$.

Therefore, the direction $V(c(t))$ is the decreasing function of t.

DARIUSZ SIERGIEJUK 4.3 VECTOR FIELDS HOMEWOR #1

27. Let $F(x,y,z)=(xe^y, y^2z^2, xyz)$ and suppose $c(t)=(x(t),y(t),z(t))$ is a flow line for F.

Find the system of differential equations that the functions $x(t), y(t)$, and $z(t)$ must satisfy.

We are given the following function,

$F(x,y,z)=(xe^y, y^2z^2, xyz)$

And our curve is $c(t)=(x(t),y(t),z(t))$ is a flow line for F.

Since c is a flow line for F. We have $c'(t)=F(c(t))$.

Which is our derivative,

$c'(t)=(x(t)e^{y(t)}, (y(t))^2(z(t))^2, x(t)y(t)z(t))$

From the above, the system of differential equations that the functions the point $x(t), y(t), z(t)$ shall meet the following criteria:

$x'(t)=x(t)e^{y(t)}$

$y'(t)=(y(t))^2(z(t))^2$

$z'(t)=x(t)y(t)z(t)$

DARIUSZ SIERGIEJUK 4.4 DIVERGENCE AND CURL HOMEWORK #1

Find the divergence of the vector fields in Exercises 1 to 4.

4. $V(x, y, z) = x^2 i + (x+y)^2 j + (x+y+z)^2 k$

Here, we have the following vector field $V(x, y, z) = x^2 \hat{i} + (x+y)^2 \hat{j} + (x+y+z)^2 \hat{k}$

We have to locate/find the divergence of the given vector field.

When $F = F_1 \hat{i} + F_2 \hat{j} + F_3 \hat{k}$, what we gonna obtain then is the divergence of F is the scalar field

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Where $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

Divergence of V is the scalar field and it is given by,

$$\operatorname{div} V = \frac{\partial}{\partial x} x^2 + \frac{\partial}{\partial y}(x+y)^2 + \frac{\partial}{\partial z}(x+y+z)^2$$

$$= 2x + 2(x+y) + 2(x+y+z)$$

$$= 2x = 2x + \boxed{2y} + 2x + \boxed{2y} + 2z$$

$$= 6x + 4y + 2z \qquad \text{Factor out 2}$$

$$= 2(3x+2y+z)$$

This way we obtained the divergence of the given vector field as $\operatorname{div} V = \boxed{2(3x+2y+z)}$

7. Sketch a few flow lines for $F(x,y) = y\hat{i}$.

Calculate $\nabla \cdot F$ and explain why your answer is consistent with your sketch.

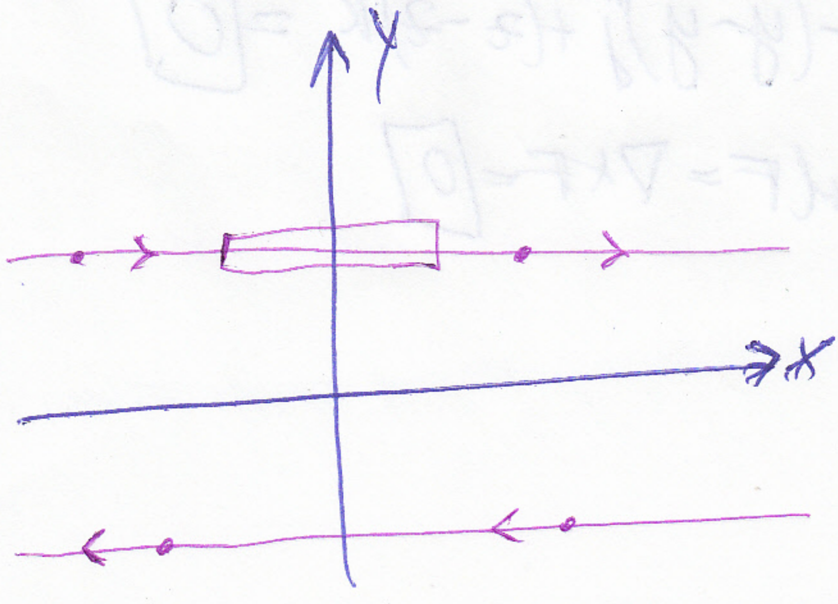
Here, we have as follows $F(x,y) = y\hat{i}$.
As stated in the exercise directions, we need to find $\nabla \cdot F$ and need to provide our sketch. In addition, the explanation for the value of $\nabla \cdot F$ and the sketch itself has to be provided.

$$\nabla \cdot F = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(y\hat{i}\right)$$

$$= \frac{\partial}{\partial x} y$$

$$\nabla \cdot F = 0$$

In the event, where F denotes some fluid, we won't observe any expansion or contraction (compression).
As a result the area of small rectangle stays the same.



Compute the curl, $\nabla \times F$, of the vector fields in Exercises 13 to 16.

14. $F(x,y,z) = yz\hat{i} + xz\hat{j} + xyk$

Here we go again with the provided vector field as

$$F(x,y,z) = yz\hat{i} + xz\hat{j} + xy\hat{k}$$

We have to calculate the curl $(\nabla \times F)$ of the above vector field.

$$\text{Curl } F = \nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix}$$

or

$$+\left(\frac{\partial}{\partial z}(yz) - \frac{\partial}{\partial x}(xy)\right)\hat{j}$$

$$= \left(\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(xz)\right)\hat{i} - \left(\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial z}(yz)\right)\hat{j}$$

$$+ \left(\frac{\partial}{\partial x}(xz) - \frac{\partial}{\partial y}(yz)\right)\hat{k} =$$

$$= (x-x)\hat{i} - (y-y)\hat{j} + (z-z)\hat{k} = \boxed{0}$$

Therefore, $\text{Curl } F = \nabla \times F = \boxed{0}$

Calculate the scalar curl of each of the vector fields in Exercises 17 to 20.

47. $F(x,y) = \sin x\, i + \cos x\, j$

In this instance, our vector field is $F(x,y) = \sin x\,\hat{i} + \cos x\,\hat{j}$

We have to calculate the scalar curl of the provided vector field.

The curl is

$$\nabla \times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin x & \cos x & 0 \end{vmatrix}$$

or

$$+\left(\frac{\partial}{\partial z}(\sin x) - \frac{\partial}{\partial x}(0)\right)\hat{j}$$

$$= \left(\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial x}(\cos x)\right)\hat{i} - \left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial z}(\sin x)\right)\hat{j}$$

$$+ \left(\frac{\partial}{\partial x}(\cos x) - \frac{\partial}{\partial y}(\sin x)\right)\hat{k}$$

$$= (-\sin x)\hat{k}$$

The scalar curl, which is the coefficient of curl is here $(-\sin x)$.

Therefore scalar curl equals to $\boxed{-\sin x}$

21. (a) Let $F(x,y,z)=(x^2, x^2y, z+zx)$.
Verify that $\nabla\cdot(\nabla\times F)=0$.

Our vector field here is as follows,

$$F(x,y,z)=(x^2, x^2y, z+zx)$$

We have to verify that $\nabla\cdot(\nabla\times F)=0$ for the given function.

$$\nabla=\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}$$

When $F=F_1\hat{i}+F_2\hat{j}+F_3\hat{k}$, the curl of F is the vector field.

$$\operatorname{curl} F=\nabla\times F=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x^2 & x^2y & z+zx\end{vmatrix}$$

$$=\left(\frac{\partial}{\partial y}(z+zx)-\frac{\partial}{\partial z}(x^2y)\right)\hat{i}-\left(\frac{\partial}{\partial x}(z+zx)-\frac{\partial}{\partial z}(x^2)\right)\hat{j}$$
$$+\left(\frac{\partial}{\partial x}(x^2y)-\frac{\partial}{\partial y}(x^2)\right)\hat{k}$$

$$=-(z)\hat{j}+(2xy)\hat{k}$$

Our curl of F is $\nabla\times F$ and equals to $-(z)\hat{j}+(2xy)k$

21.(a) CONTINUATION

Now we have to compute divergence of curl F, which is div curl F = $\nabla\cdot(\nabla\times F)$.

$$\nabla\cdot(\nabla\times F)=\nabla\cdot(0,-z,2xy)$$

$$=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\cdot\left((0)\hat{i}-(z)\hat{j}+(2xy)\hat{k}\right)$$

$$=\boxed{0}$$

21.(b) Can there exist a function $f:\mathbb{R}^3\to\mathbb{R}$ such that $F=\nabla f$? Explain.

No, since $\nabla\times F\neq 0$.

Rationale

$$\nabla\times F=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ x^2 & x^2y & z+zx\end{vmatrix}$$

$$=\left(\frac{\partial}{\partial y}(z+zx)-\frac{\partial}{\partial z}(x^2y)\right)\hat{i}-\left(\frac{\partial}{\partial x}(z+zx)-\frac{\partial}{\partial z}(x^2)\right)\hat{j}$$
$$+\left(\frac{\partial}{\partial x}(x^2y)-\frac{\partial}{\partial y}(x^2)\right)\hat{k}$$

$$\nabla\times F=-(z)\hat{j}+(2xy)k\neq 0$$

23. Let $F(x,y,z) = (e^{xz}, \sin(xy), x^5y^3z^2)$.

(a) Find the divergence of F.

Here we have the following function

$F(x,y,z) = (e^{xz}, \sin(xy), x^5y^3z^2)$

When $F(x,y,z) = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$ the divergence of F is gonna be the scalar field,

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

Now, $\operatorname{div} F = \nabla \cdot F$

$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$= \frac{\partial}{\partial x}\left(e^{xz}\right) + \frac{\partial}{\partial y}\left(\sin(xy)\right) + \frac{\partial}{\partial z}\left(x^5y^3z^2\right)$$

$$= ze^{xz} + x\cos(xy) + 2x^5y^3z$$

Here, the divergence of F is $\operatorname{div} F = \underline{ze^{xz} + x\cos(xy) + 2x^5y^3z}$.

23. Let $F(x,y,z) = (e^{xz}, \sin(xy), x^5y^3z^2)$.

(b) Find the curl of F.

Calculating the curl of F,

$$\text{curl } F = \nabla \times F$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ e^{xz} & \sin(xy) & x^5y^3z^2 \end{vmatrix}$$

OR

$$+\left(\frac{\partial}{\partial z}(e^{xz}) - \frac{\partial}{\partial x}(x^5y^3z^2)\right)\hat{j}$$

$$= \left(\frac{\partial}{\partial y}(x^5y^3z^2) - \frac{\partial}{\partial z}(\sin(xy))\right)\hat{i} - \left(\frac{\partial}{\partial x}(x^5y^3z^2) - \frac{\partial}{\partial z}(e^{xz})\right)\hat{j}$$

$$+ \left(\frac{\partial}{\partial x}(\sin(xy)) - \frac{\partial}{\partial y}(e^{xz})\right)k$$

$$= (3x^5y^2z^2)\hat{i} - (5x^4y^3z^2 - xe^{xz})\hat{j} + (y\cos(xy))\hat{k}$$

+

Thus, we obtained the curl of F which in this case is

$$\text{curl } F = (3x^5y^2z^2)\hat{i} + (xe^{xz} - 5x^4y^3z^2)\hat{j} + (y\cos(xy))\hat{k}$$

26. Suppose $f, g, h: \mathbb{R} \longrightarrow \mathbb{R}$ are differentiable.
Show that the vector field $F(x,y,z) = (f(x), g(y), h(z))$ is irrational.

Considering the function $F(x,y,z) = (f(x), g(y), h(z))$ is differentiable.

Generally, $F(x,y,z) = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$ the divergence of F is the vector field, curl $F = \nabla \times F$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

A vector field is called irrational if curl $F = 0$.

Calculating curl F,

curl $F = \nabla \times F$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

DARIUSZ SIERGIEJUK 4.4 DIVERGENCE AND CURL HOMEWORK #1 21

26. CONTINUATION

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f(x) & g(y) & h(z) \end{vmatrix}$$

as opposed to

$$-\left(\frac{\partial h(z)}{\partial x} - \frac{\partial f(x)}{\partial z}\right)\hat{j}$$

flipped

$$\text{curl } F = \left(\frac{\partial h(z)}{\partial y} - \frac{\partial g(y)}{\partial z}\right)\hat{i} + \left(\frac{\partial f(x)}{\partial z} - \frac{\partial h(z)}{\partial x}\right)\hat{j} + \left(\frac{\partial g(y)}{\partial x} - \frac{\partial f(x)}{\partial y}\right)k$$

As $h(z)$ is a function of z not of x and y, $g(y)$ is a function of y not of x and z and $f(x)$ is a function of x not of y and z,

$$\text{curl } F = (0)i + (0)j + (0)k$$

$$= 0$$

Hence, it is proved/shown that $F(x,y,z) = (f(x), g(x), h(x))$ is irrotational.

27. Suppose $f, g, h: \mathbb{R}^2 \mapsto \mathbb{R}$ are differentiable.
Show that the vector field $F(x,y,z) = (f(y,z), g(x,z), h(x,y))$ has zero divergence.

Here we have the following function $F(x,y,z) = (f(y,z), g(x,z), h(x,y))$ which is differentiable.

Generally, when $F(x,y,z) = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$ the divergence of F is the scalar field,

$$\operatorname{div} F = \nabla \cdot F = \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

In order to calculate the divergence of F we proceed as follows,

$$\operatorname{div} F = \nabla \cdot F$$
$$= \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z}$$

$$\operatorname{div} F = \frac{\partial}{\partial x}(f(y,z)) + \frac{\partial}{\partial y}(g(x,z)) + \frac{\partial}{\partial z}(h(x,y))$$

As $f(y,z)$, $g(x,z)$, and $h(x,y)$ are not the functions of x, y, and z respectively.

$$\operatorname{div} F = 0 + 0 + 0 = \boxed{0}$$

Consequently, we have shown/proved that the given in this exercise vector field $F(x,y,z) = (f(y,z), g(x,z), h(x,y))$ has no (zero) divergence.