Fortino Flores Hw#2 2/24/17

Excellent!

2/2

24) (a) curl(grad f) = Vector field

(b) meaning less a curl can only be taken by a

(c) scalar field divergence of a VF = SF

(d) meaningless = div of a SF cannot be taken

(e) meaningless = div of a SF cannot be taken

(f) meaningless = curl can only be taken by a VF

25

(a) meaningless ~~curl can only be taken~~ gradient can only be taken SF

(b) meaningless grad can be taken by SF

(c) meaningless grad can only be taken by SF

(d) VF

(e) meaningless curl can only be taken by a VF

(f) SF

30 Verify $\nabla \times (\nabla f) = 0$ curl of a gradient

$f(x,y,z) = xy + yz + xz$

$$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} & \frac{\partial f}{\partial z} \end{vmatrix} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y+z & x+z & y+x \end{vmatrix} = i\left(\frac{\partial}{\partial y}(y+x) - \frac{\partial}{\partial z}(x+z)\right) - j\left(\frac{\partial}{\partial x} y+x - \frac{\partial}{\partial z} y+z\right) + k\left(\frac{\partial}{\partial x} x+z - \frac{\partial}{\partial y} y+z\right)$$

$$= i(x-x) - j(y-y) + k(z-z)$$

$$= i(0) - j(0) + k(0)$$

$$= \vec{0}$$

(33) If F were a vector field it would satisfy curl $U = 0$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y\sin x + x & \cos x + x\sin y & 0 \end{vmatrix}$$

$F = y(\cos x)i + x(\sin y)j + (0)k$

$(-y\sin x) + x + 0$

$\frac{\partial f}{\partial y} = \cos x + x\sin y$

$$i\left(\frac{\partial}{\partial y}0 - \frac{\partial}{\partial z}(\cos x + x\sin y)\right) - j\left(\frac{\partial}{\partial x}0 - \frac{\partial}{\partial z}(-y\sin x + x)\right) + k\left(\frac{\partial}{\partial x}(\cos x + x\sin y) - \frac{\partial}{\partial y}(-y\sin x + x)\right)$$

$$i(0) - j(0) + k(-\sin x + \sin y) = \boxed{(-\sin x + \sin y)k \neq 0}$$

(34) $F = (x^2 + y^2)i - 2xyj$

$$\text{curl } F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x - 2y & 2y - 2x & 0 \end{vmatrix}$$

$\frac{\partial f}{\partial x} = 2x - 2y$

$\frac{\partial f}{\partial y} = 2y - 2x$

$$= i\left(\frac{\partial}{\partial y}0 - \frac{\partial}{\partial z}(2y - 2x)\right) - j\left(\frac{\partial}{\partial x}0 - \frac{\partial}{\partial z}(2x - 2y)\right) + k\left(\frac{\partial}{\partial x}(2y - 2x) - \frac{\partial}{\partial y}(2x - 2y)\right)$$

$$= i(0) - j(0) + k(-2 - 2) = \boxed{-4k \neq 0}$$

a) $\nabla(1/r) = -r/r^3,\ r \neq 0$ # 38.

$= \frac{\partial(\frac{1}{r})}{\partial x} i + \frac{\partial(\frac{1}{r})}{\partial y} j + \frac{\partial(\frac{1}{r})}{\partial z} k = -\frac{x}{r^3} i - \frac{y}{r^3} j - \frac{z}{r^3} k = -\frac{r}{r^3}$

$\nabla(r^n) = n r^{n-2} r$

$= \frac{\partial}{\partial x} r^n = \frac{\partial}{\partial x}(x^2+y^2+z^2)^{n/2} = 2x\frac{n}{2}(x^2+y^2+z^2)^{n/2-1}$

$= n x r^{n-2}$

$\frac{\partial}{\partial y} r^n = n y r^{n-2}$, $\frac{\partial}{\partial z} r^n = n z r^{n-2}$

$= \left(\frac{\partial}{\partial x}(r^n), \frac{\partial}{\partial y}(r^n), \frac{\partial}{\partial z}(r^n)\right) = (n x r^{n-2}, n y r^{n-2}, n z r^{n-2}) = n r^{n-2} r$

Hence $\nabla(\log r) = \frac{r}{r^2}$

B)

$\nabla^2 f = \nabla \cdot (\nabla f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

$\nabla(\frac{1}{r}) = -\frac{r}{r^3}$

$\nabla^2(\frac{1}{r}) = \frac{-x}{(x+y+z)^3} - \frac{y}{(x+y+z)^3} - \frac{z}{(x+y+z)^3} = 0$

$\nabla^2 r^n = \nabla \cdot \nabla r^n = \nabla \cdot n r^{n-2} r = n(n+1) r^{n-2}$

c) $\nabla \cdot (r/r^3) = 0$

$= \frac{\partial(\frac{r}{r^3})}{\partial x} + \frac{\partial}{\partial y}\left(\frac{r}{r^3}\right) + \frac{\partial}{\partial z}\left(\frac{r}{r^3}\right) = 0$

$\nabla \cdot (r^n r) = (n r^{n-2} r) r$

$= n r^{2n-4} r^2,\ \nabla = (n+3) r^n$

D) $\nabla \times r = 0$ $\quad r = (x, y, z)$

$= \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 1 & 1 & 1 \end{vmatrix} = i(0-0) + j(0-0) + k(0-0) = \vec{0}$

hence $\nabla \times (r^n r) = \vec{0}$

1) $f(x,y,z) = y$ $\quad c(t) = (0,0,t) \quad 0 \le t \le 1$

$\boxed{\#9}$

$\boxed{pt\ 2}$

$$\|c'(t)\| = \sqrt{(0)^2 + (0)^2 + \left(\frac{dt}{dt}\right)^2}$$

$$= \sqrt{(1)^2}$$

$$= 1$$

$f(x,y,z) = y = 0$ ✓

$$\int_0^1 0 \cdot 1 \, dt = \boxed{0}$$

10) Evaluate the following Path Integrals $\int_c f(x,y,z)\, ds$ where

a) $f(x,y,z) = x+y+z$ and $c: t \to (\sin t, \cos t, t,) \quad t \in [0, 2\pi]$

$$\|c'(t)\| = \sqrt{\left(\frac{d}{dt}(\sin t)\right)^2 + \left(\frac{d(\cos t)}{dt}\right)^2 + \left(\frac{dt}{dt}\right)^2} = \sqrt{\cos^2 t + \sin^2 t + 1} = \sqrt{2}$$

$f(x,y,z) = x + y + z = \sin t + \cos t + t$

$$\int_0^{2\pi} (\sin t + \cos t + t) \cdot \sqrt{2}\, dt = \sqrt{2} \int_0^{2\pi} \sin t + \cos t + t \, dt$$

$$= \sqrt{2} \left[-\cos t + \sin t + \frac{t^2}{2} \right]_0^{2\pi}$$

$-1 + 0 + 0$

$-1 + 0 + \frac{(2\pi)^2}{2}$

$-1 + \frac{4\pi^2}{2}$

$\cancel{-1} + 2\pi^2 \cancel{+1} =$

$\boxed{2\pi^2 \cdot \sqrt{2}}$ ✓

b) $f(x,y,z) = \cos z \quad c: t \to (\sin t, \cos t, t) \quad t \in [0, 2\pi]$

$\|c'(t)\| = \sqrt{2}$

$f(x,y,z) = \cos(t)$

$\int_0^{2\pi} \cos t \sqrt{2}\, dt = \sqrt{2} \int_0^{2\pi} \cos t\, dt = -\sqrt{2} \sin t \Big|_0^{2\pi}$

$= -\sqrt{2} \sin(2\pi)$

$= \boxed{0}$

12) a) $f(x,y,z) = x \cos z \quad c: t \to ti + t^2 j \quad t \in [0,1]$

$\|c'(t)\| = \sqrt{\left(\frac{dt}{dt}\right)^2 + \left(\frac{d(t^2)}{dt}\right)^2 + 0}$

$= \sqrt{1 + 2t}$

$f(x,y,z) = t\cos(0)$

$= \int_0^1 t\cos(0)\sqrt{1+4t^2}\, dt = \frac{1}{8} \frac{(1+4t^2)^{3/2}}{3/2} \Big|_0^1$

$= \boxed{\frac{5^{3/2} - 1}{12}}$

b) $f(x,y,z) = (x+y)/(y+z) \quad c: t \to (t, \frac{2}{3}t^{3/2}, t), t \in [1,2]$

$\|c'(t)\| = \sqrt{\left(\frac{d(t)}{dt}\right)^2 + \left(\frac{d(\frac{2}{3}t^{3/2})}{dt}\right)^2 + \left(\frac{d(t)}{dt}\right)} = \sqrt{2+t}$

$f(x,y,z) = \left(\frac{t + \frac{2}{3}t^{3/2}}{\frac{2}{3}t^{3/2} + t}\right)$

$\int \frac{t + \frac{2}{3}t^{3/2}}{\frac{2}{3}t^{3/2} + t} \sqrt{2+t}\, dt = \frac{(2+t)^{3/2}}{3/2} \Big|_1^2 = \boxed{\frac{8 - 3^{3/2}}{3/2}}$

6) a) $S_N := \sum_{i=1}^{N} f(x(t_i^*), y(t_i^*))(s_i - s_{i-1})$

Where we have $(s_i - s_{i-1}) = \int_{t_i}^{t_{i+1}} \|c'(t)\| \, dt$ then

$$\sum_{i=1}^{N} (s_i - s_{i-1}) = L(c)$$

and so the quotient

$$\frac{\sum_{i=1}^{N} f(x(t_i^*), y(t_i^*))(s_i - s_{i-1})}{\sum_{i=1}^{N} (s_i - s_{i-1})} = \boxed{\frac{\sum_{i=1}^{N} f(x(t_i^*), y(t_i^*))(s_{i-1})}{L(c)}}$$

b) $$\frac{\int_0^{2\pi} (x^2+y^2+z^2)\|c'(t)\| \, dt}{\int_0^{2\pi} \|c'(t)\| \, dt} = \frac{\int_a^b (\cos^2(t) + \sin^2(t) + t^2)\sqrt{2} \, dt}{\int_0^{2\pi} \sqrt{2} \, dt}$$

$$= \frac{\int_0^{2\pi} (1+t^2)\sqrt{2} \, dt}{\int_0^{2\pi} \sqrt{2} \, dt} = \frac{\sqrt{2}\left[t + \frac{t^3}{3}\right]\Big|_0^{2\pi}}{\sqrt{2}(2\pi)}$$

$$= \frac{\frac{2\sqrt{2}\pi}{3}(3+4\pi^2)}{\sqrt{2}(2\pi)} = \frac{1}{3}(3+4\pi^2) = \boxed{1 + \frac{4\pi^2}{3}}$$

c) a) $\frac{2\pi^2 \cdot \sqrt{2}}{\sqrt{2}(2\pi)} = \boxed{\pi}$

b) $\frac{0}{\sqrt{2}} = \boxed{0}$

18) a) what is the total mass of the wire? $\theta \to (0, a\sin\theta, a\cos\theta)$

$$\int_0^{\pi} \|c'(t)\|\, dt = \int_0^{\pi} \sqrt{\left(\frac{d0}{d\theta}\right)^2 + \left(\frac{a\sin\theta\, d}{d\theta}\right)^2 + \left(\frac{a\cos\theta\, d}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{\pi} \sqrt{0 + a\cos^2\theta + a\sin^2\theta}\, d\theta$$

$$= \int_0^{\pi} \sqrt{2a}\, d\theta = \frac{2}{3}(2a)^{3/2} \Big|_0^{\pi} = \boxed{\frac{2}{3} 2\pi^{3/2} \text{ grams}}$$

b) $x_c = \frac{1}{l}\int_c x\, ds = 0$

Center is at $(0, 2a)$

$y_c = \frac{1}{l}\int_c y\, ds = \boxed{2a}$

19) a) Find $l(c)$ $c(t) = (t^2, t, 3)$ for $t \in [0,1]$

$$l(c) = \int_c \|c'(t)\|\, dt = \int \sqrt{\left(\frac{d}{dt}t^2\right)^2 + \left(\frac{d}{dt}(t)\right)^2 + \left(\frac{d}{dt}(3)\right)^2}\, dt$$

$$= \boxed{\frac{2\sqrt{5} + \log(2+\sqrt{5})}{4}}$$

b) Average y-coordinate

$$y_c = \frac{1}{l}\int_c y\, ds = \boxed{\frac{(5\sqrt{5}-1)}{[6\sqrt{5} + 3\log(2+\sqrt{5})]}}$$