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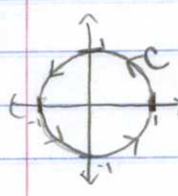
Professor Park
Math 255
3/3/17

Excellent! 2/2

Homework #3, 7.2

2.) $F = y^2 i + 2xy j$

$$\int_c F \cdot ds = \int_a^b F(c(t)) \cdot c'(t)\, dt$$



$c:[0,2\pi] \to \mathbb{R}^2,\ t \mapsto (\overset{x}{\cos t}, \overset{y}{\sin t}) \quad 0 \le t \le 2\pi$

$$\int_0^{2\pi} (\sin^2 t)\frac{dx}{dt} + (2\cos t \sin t)\frac{dy}{dt}\ dt$$

$$= \int_0^{2\pi} (-\sin^3 t + 2\cos^2 t \sin t)\, dt$$

$$\int (-\sin^2 t + 2\cos^2 t)(\sin t)\, dt$$

$u = \cos t$
$du = -\sin t\, dt$
$-du = \sin t\, dt$

$$-\int (-(1-u^2) + 2u^2)\, du$$

$$\int (-1 + u^2 + 2u^2)\, du$$

$$-\left(-u + \frac{u^3}{3} + \frac{2u^3}{3}\right)$$

$$-\left(-\cos t + \frac{\cos^3 t}{3} + \frac{2\cos^3 t}{3}\right)\Big|_0^{2\pi}$$

$$-\left[\left(-1 + \tfrac{1}{3} + \tfrac{2}{3}\right) - \left(-1 + \tfrac{1}{3} + \tfrac{2}{3}\right)\right]$$

$$= 0$$

$$\boxed{\int_C F \cdot ds = 0}$$

4.) a.) $\int_c x\,dy - y\,dx$, $c(t) = (\cos t, \sin t)$, $0 \le t \le 2\pi$

$$\int_c -y\,dx + x\,dy$$

$c'(t) = (-\sin t, \cos t)$

$F(c(t)) = (-\sin t, \cos t)$

$$\int_0^{2\pi} (\sin^2 t + \cos^2 t)\,dt$$

$$\int_0^{2\pi} 1\,dt$$

$$t\Big|_0^{2\pi} = \boxed{2\pi}$$

b.) $\int_c x\,dx + y\,dy$, $c(t) = (\cos(\pi t), \sin(\pi t))$, $0 \le t \le 2$

$c'(t) = (-\pi\sin(\pi t), \pi\cos(\pi t))$

$F(c(t)) = (\cos \pi t, \sin \pi t)$

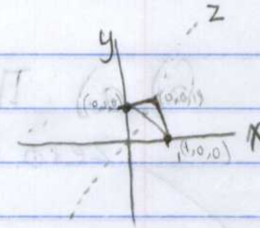
$$\int_0^2 \left(\cos(\pi t)(-\pi\sin(\pi t)) + \sin(\pi t)\pi\cos(\pi t)\right)dt$$

$$\int_0^2 (-\cos\pi t\sin\pi t + \cos\pi t\sin\pi t)\,\pi\,dt$$

$$\int_0^2 0\,dt$$

$$\boxed{0}$$

(c) $\int_C yz\,dx + xz\,dy + xy\,dz$, where c is



$$c:[0,3]\to\mathbb{R}^3,\ t\mapsto \begin{cases} (t,1-t,0) & 0\le t\le 1 \\ (2-t,0,t-1) & 1\le t\le 2 \\ (0,t-2,3-t) & 2\le t\le 3 \end{cases}$$

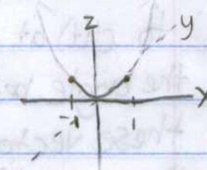
$$\int_C yz\,dx + xz\,dy + xy\,dz = \int_0^1 (1-t)(0)\tfrac{dx}{dt} + (t)(0)\tfrac{dy}{dt} + (t)(1-t)\tfrac{dz}{dt}\,dt$$

$$+ \int_1^2 (0)(t-1)\tfrac{dx}{dt} + (2-t)(t-1)\tfrac{dy}{dt} + (2-t)(0)\tfrac{dz}{dt}\,dt$$

$$+ \int_2^3 (t-2)(3-t)\tfrac{dx}{dt} + (0)(3-t)\tfrac{dy}{dt} + (0)(t-2)\tfrac{dz}{dt}\,dt$$

$$= \int_0^1 0\,dt + \int_1^2 0\,dt + \int_2^3 0\,dt = \boxed{0}$$

d) $\int_c x^2\,dx - xy\,dy + dz$, where c is



$$c:[-1,1]\to\mathbb{R}^3,\ t\mapsto(t,0,t^2)\quad -1\le t\le 1$$

$F(c(t)) = (t^2,0,1)$

$c'(t) = (1,0,2t)$

$$\int_c x^2\,dx - xy\,dy + dz = \int_{-1}^1 \left((t^2)(1) - (0)(t)(0) + (1)(2t)\right)dt$$

$$= \int_{-1}^1 t^2 + 2t\,dt$$

$$\frac{t^3}{3} + t^2 \Big|_{-1}^{1}$$

$$\left(\tfrac{1}{3}+1\right) - \left(-\tfrac{1}{3}+1\right)$$

$$\tfrac{4}{3} - \tfrac{2}{3} = \tfrac{2}{3}$$

$$\boxed{\frac{2}{3}}$$

6) a.) The dot product of two non-zero vectors is zero if and only if the vectors are perpendicular

$$\int_C F \cdot ds = \int_a^b (F(c(t)) \cdot c'(t))\, dt$$

If $F(c(t))$ is perpendicular to $c'(t)$, then $F(c(t)) \cdot c'(t)$ must be zero.

proof: Let u and v be non-zero, orthogonal vectors.

$u \cdot v = \|u\|\|v\| \cos \frac{\pi}{2}$
$= \|u\|\|v\| (0)$
$= 0$

Assume $u \cdot v = 0$ Then $\|u\|\|v\| \cos\theta = 0$

$\cos\theta = \frac{0}{\|u\|\|v\|} = 0$

Part b.) $\int_C F \cdot ds = \int_a^b (F(c(t)) \cdot c'(t))\, dt$

$F(c(t)) = \lambda(t) c'(t)$

Since F is parallel to $c'(t)$ at $c(t)$, the angle between these vectors is 0 or 180 degrees. $F(c(t))$ and $c'(t)$ are scalar.

$\int \lambda(t) c'(t) \cdot c'(t)\, dt$

$\int \lambda(t) \|c'(t)\|^2\, dt$

So $\int_C F \cdot ds = \int_C \|F\|\, ds$

7) <u>Cauchy-Schwarz Inequality</u>

$$|a \cdot b| \le \|a\| \|b\|$$

$$\int_C F \cdot ds = \int_a^b F(c(t)) \cdot c'(t)\, dt$$

$$\text{So } \left| \int_C F \cdot ds \right| = \left| \int_a^b F(c(t)) \cdot c'(t)\, dt \right|$$

<u>Note:</u> $\left| \int_a^b F(t)\,dt \right|$ is equal to $\int_a^b |F(t)|\,dt$ if the function $F(t)$ is either entirely positive or entirely negative. (or zero)

$\left| \int_a^b F(t)\,dt \right|$ is less than $\int_a^b |F(t)|\,dt$ otherwise.

$$\text{So } \left| \int_a^b F(c(t)) \cdot c'(t)\,dt \right| \le \int_a^b |F(c(t)) \cdot c'(t)|\, dt$$

$$\le \int_a^b \|F(c(t))\| \|c'(t)\|\, dt$$

Since $\|F\| \le M$,

$$\le \int_a^b M \|c'(t)\|\, dt$$

$$= M \int_a^b \|c'(t)\|\, dt$$

$$= M\ell$$

8.) Evaluate $\int_c F \cdot ds$, where $F(x,y,z) = yi + 2xj + yk$

$c(t) = ti + t^2 j + t^3 k \ , \ 0 \le t \le 1.$

$F = (y, 2x, y)$

$c(t) = (t, t^2, t^3)$

$c'(t) = (1, 2t, 3t^2)$

$F(c(t)) = (t^2, 2t, t^2)$

$$\int_c F \cdot ds = \int_0^1 (t^2 + 4t^2 + 3t^4)\, dt$$

$$\frac{t^3}{3} + \frac{4t^3}{3} + \frac{3t^5}{5} \Big|_0^1$$

$$\left(\frac{1}{3} + \frac{4}{3} + \frac{3}{5}\right) - (0)$$

$$\frac{5}{15} + \frac{20}{15} + \frac{9}{15} = \boxed{\frac{34}{15} = 2\tfrac{4}{15}}$$

No need. I don't care. I don't know who actually cares

11, 13, 17, 18

11.) $F(x,y) = xi + yi$

$c(t) = (\cos^3 t, \sin^3 t) \ , \ 0 \le t \le 2\pi$

$(\cos())^3$

$3(\cos())^2(-\sin t)$

$c'(t) = (-3\cos^2 t \sin t, \ 3\sin^2 t \cos t)$

$F(c(t)) = (\cos^3 t)\, i + (\sin^3 t)\, j$

$$\int_a^b F(c(t)) \cdot c'(t)\, dt = \int_0^{2\pi} -3\cos^5 t \sin t + 3\sin^5 t \cos t \, dt$$

$$= 3\int_0^{2\pi} -\cos^5 t \sin t + \sin^5 t \cos t \, dt$$

$$= 3\left[\int_0^{2\pi} -\cos^5 t \sin t \, dt + \int_0^{2\pi} \sin^5 t \cos t \, dt\right]$$

$u = \cos t$ $\qquad$ $u = \sin t$

$\int u^5 du$ $\qquad$ $\int u^5 dt$

$\frac{u^6}{6}$ $\qquad$ $\frac{u^6}{6}$

$$= 3\left[\frac{1}{3}\cos^6 t\right]\Big|_0^{2\pi}$$

$$\left(\cos^6(2\pi) - \cos^6(0)\right)$$

$$1 - 1$$

$$\boxed{0}$$

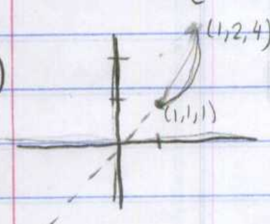
13.) Let $c(t)$ be a path and T the unit tangent vector. What is $\int_c T \cdot ds$?

If T is the unit tangent vector, then

$$\int_c T \cdot ds = \int_c \|c'(t)\|\, dt$$

Since $\int_c \|c'(t)\|\, dt$ is the length of the path c, $\int_c T \cdot ds$ must be the length of the path c.

17.)



$c: [1,2] \to \mathbb{R}^3,\ t \mapsto (1, t, t^2)\ ,\ 1 \le t \le 2$

$$F(x,y,z) = (2xyz,\ x^2 z,\ x^2 y)$$

$$c'(t) = (0, 1, 2t)$$

$$F(c(t)) = (2t^3, t^2, t)$$

$$\int_a^b F(c(t)) \cdot c'(t)\, dt = \int_1^2 (t^2 + 2t^2)\, dt$$

$$\frac{t^3}{3} + \frac{2t^3}{3}$$

$$t^3 \Big|_1^2$$

$$2^3 - 1^3$$

$$8 - 1$$

$$\boxed{7}$$

18) $\nabla f(x,y,z) = \left(2xyze^{x^2}\right) i + \left(ze^{x^2}\right) j + \left(ye^{x^2}\right) k$

Since $\frac{\partial}{\partial x}\left(yze^{x^2}+c\right) = 2xyze^{x^2}$

$\frac{\partial}{\partial y}\left(yze^{x^2}+c\right) = ze^{x^2}$

$\frac{\partial}{\partial z}\left(yze^{x^2}+c\right) = ye^{x^2}$,

$f(x,y,z) = yze^{x^2} + c$

Since $f(0,0,0)=5$, $c=5$

So $f(x,y,z) = yze^{x^2} + 5$

$f(1,1,2) = 1 \cdot 2 \cdot e^{1^2} + 5$

$= \boxed{\begin{matrix} 2e+5 \\ \approx 10.44 \end{matrix}}$