Midterm Exam Review Sheet
MATH 25500 Section 01
Exam date and time: 24th March 2017, 14:10–15:25

Instructions: In the exam, you may use your textbook or notes, but not any electronic device. Talking to other students will not be allowed. Answers without justifications and/or calculation steps may receive no score. 10 points each unless stated otherwise.

1. Let $f : \mathbb{R}^3 \to \mathbb{R}$ be a C^1-function and $\overrightarrow{F}$ a C^1-vector field on $\mathbb{R}^3$. Show that $\nabla \times (f\overrightarrow{F}) = f\nabla \times \overrightarrow{F} + \nabla f \times \overrightarrow{F}$.

From here to the #8 of this exam, $\overrightarrow{F}(x, y, z) = (x, y, z^2)$ is a vector field on $\mathbb{R}^3$.

2. Find $\nabla \cdot \overrightarrow{F}$ and $\nabla \times \overrightarrow{F}$.

3. Let $\alpha(t) = (\cos t, \sin t, t)$ be a curve in $\mathbb{R}^3$ defined on $[0, 2\pi]$. Calculate $\int_\alpha \overrightarrow{F} \cdot d\overrightarrow{r}$.

From here to the end of this exam, $\Phi : D \subset \mathbb{R}^2 \to \mathbb{R}^3$ defined by

$$(u, v) \mapsto (\cos u, \sin u, v)$$

is a parametrization of a surface S, where $D = [0, 2\pi] \times [0, 1]$.

4. Find all points $(u, v) \in D$ such that the surface S is regular at $\Phi(u, v)$.

5. Find the area of S by using the surface integral.

6. Evaluate $\iint_S z^2 dS$.

7. Compute the surface integral $\iint_S \overrightarrow{F} \cdot d\overrightarrow{S}$.

8. (20 points) Calculate the Gauss curvature and the mean curvature at each point $p \in S$.

9. The temperature distribution in $\mathbb{R}^3$ is given by $T(x, y, z) = 3x^2 + 3z^2$. Compute the heat flux across the surface $x^2 + z^2 = 2$ and $0 \le y \le 2$. Here the heat flux $\overrightarrow{F}$ is defined by $\overrightarrow{F} := -\nabla T$.