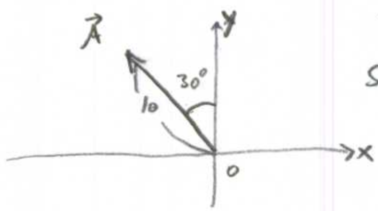
Midterm Examination 1
MTH 13 Section E01
21 February 2017 18:00–19:50

Instructions: Please answer the following and be sure to show your work or support your answer. You are not allowed to use the textbook, workbook, or notes. You cannot talk to other sudents. You can use your calculator.

1. The vector $\overrightarrow{A}$ of length 10 is in the second-quadrant. The angle between $\overrightarrow{A}$ and the y-axis is 30°. Resolve the vector $\overrightarrow{A}$ (i.e. write $\vec{\mathbf{A}}$ into the sum $\vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y$).



The standard position angle of $\vec{A}$ is 120°

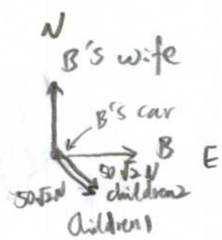
So

$$A_x = 10 \cos 120° = -10 \sin 30° = -10 \cdot \tfrac{1}{2} = -5$$
$$A_y = 10 \sin 120° = +10 \cos 30° = 10 \tfrac{\sqrt{3}}{2} = 5\sqrt{3}.$$

If we denote by $\vec{e}_x$, $\vec{e}_y$ length 1 vectors in the positive x- and y-axis direction,

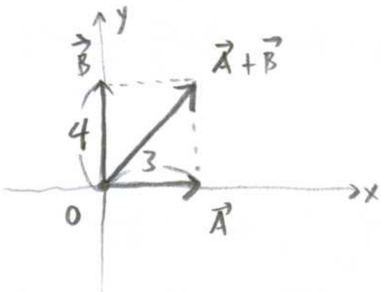
$$\underline{\vec{A} = -5\vec{e}_x + 5\sqrt{3}\,\vec{e}_y.}$$

2. B's car is in mud. B, his wife, and their two children are trying to pull the car from it. B is applying 100 Newton of force to the East, and his wife 100 Newton to the North. Each children is pulling the car in $50\sqrt{2}$ Newton of force to the Southeast. What is the total force applied to the car?



Note that the sum of forces applied by B's two children is $100\sqrt{2}$ N to the Southeast. It is resolved into a 100N of force to the south and another 100N of force to the East. The 100N to the South cancels the force applied by B's wife, and the 100N to the East adds to the 100 N of force applied by B. Hence the total force is <u>200 N to the East</u>.

3. Add two vectors $\vec{A}$ an $\vec{B}$ where the lengths of these vectors are $A = 3$ and $B = 4$. The angles in standard position of these vectors are $0°$ and $90°$, respectively. Give your answer in "length $\angle$ angle" form. You may use $53.13° = \tan^{-1}\left(\frac{4}{3}\right)$.



The length of $\vec{A}+\vec{B}$ is

$$\sqrt{A^2 + B^2} = \sqrt{3^2+4^2} = \sqrt{25} = 5.$$

where A is the length of $\vec{A}$ and B the length of $\vec{B}$

The angle (in standard position) is

$$\tan^{-1}\left(\frac{(\vec{A}+\vec{B})_y}{(\vec{A}+\vec{B})_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°. \text{ So } \underline{5 \angle 53.13°}.$$

4. Add three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ where the lengths of these vectors are $A = 10$, $B = 20$, and $C = 30$. The angles in standard position of these vectors are $0°$, $120°$, and $225°$, respectively. Give your answer in "length $\angle$ angle" form. You may use $\tan^{-1}\left(1 - \sqrt{\frac{2}{3}}\right) = 10.4°$ and $\sqrt{12 - 3\sqrt{6}} = 2.156$

	x - component	y - Component
$\vec{A}$	$10\cos 0° = 10$	$10\sin 0° = 0$
$\vec{B}$	$20\cos 120° = -10$	$20\sin 120° = 10\sqrt{3}$
$\vec{C}$	$30\cos 225° = -15\sqrt{2}$	$30\sin 225° = -15\sqrt{2}$
$\vec{A}+\vec{B}+\vec{C}$	$-15\sqrt{2}$	$10\sqrt{3} - 15\sqrt{2}$

$\cos 120° = -\sin 30° = -\frac{1}{2}$
$\sin 120° = \cos 30° = \frac{\sqrt{3}}{2}$
$\cos 225° = -\cos 45° = -\sqrt{2}/2$
$\sin 225° = -\sin 45° = -\sqrt{2}/2$

Angle : $\tan^{-1}\left(\frac{(\vec{A}+\vec{B}+\vec{C})_y}{(\vec{A}+\vec{B}+\vec{C})_x}\right) = \tan^{-1}\left(\frac{10\sqrt{3}-15\sqrt{2}}{-15\sqrt{2}}\right) = 10.4° + 180°$ (Since x-comp: -21.21, y-comp: -3.89. The angle is in the 3rd quadrant.)

Length $= \sqrt{(\vec{A}+\vec{B}+\vec{C})_x^2 + (\vec{A}+\vec{B}+\vec{C})_y^2} = \sqrt{225 \cdot 2 + 300 - 300\sqrt{6} + 225 \cdot 2}$

$= \sqrt{1200 - 300\sqrt{6}} = 10\sqrt{12 - 3\sqrt{6}} = 21.56$

Answer: $\underline{21.56 \angle 190.4°}$

5. Find values of x and y that satisfies the following equation: $9 - i = xi + 1 - y$.

Real part $9 = 1 - y \Rightarrow y = -8$

Imaginary part $-1 = x \Rightarrow x = -1.$

$x = -1, \; y = -8$

6. Express the following expression in the form of $a + bi$.

$$\frac{i}{1+i} - \frac{8-i}{2+i}$$

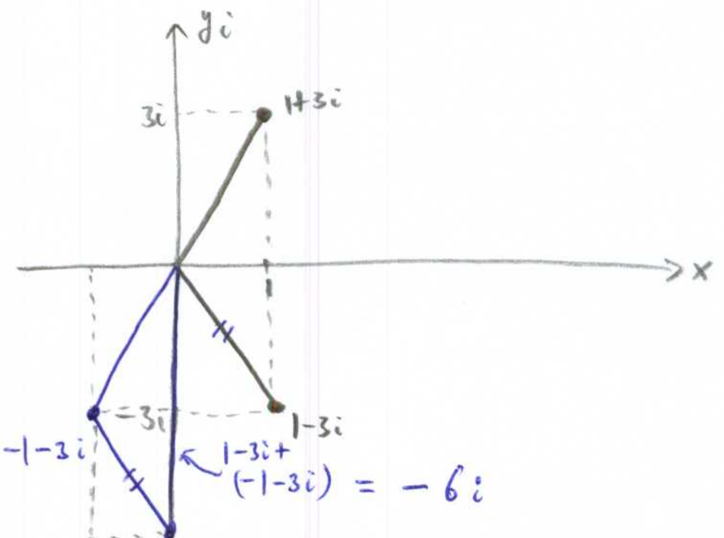
$$\frac{i(2+i)}{(1+i)(2+i)} - \frac{(8-i)(1+i)}{(2+i)(1+i)} = \frac{2i - 1 - 8 + i - 8i - 1}{(1+i)(2+i)}$$

$$= \frac{-10-5i}{1+3i} = \frac{(-10-5i)(1-3i)}{(1+3i)(1-3i)} = \frac{-25+25i}{10}$$

$$= -\frac{5}{2} + \frac{5}{2}i$$

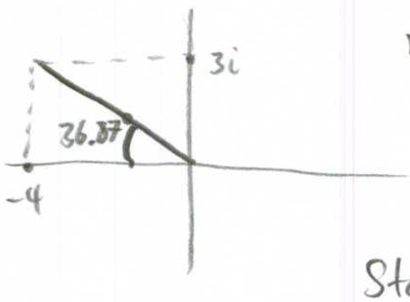
Answer

7. Subtract $1 + 3i$ from $1 - 3i$ graphically.



8. Write $-4 + 3i$ in polar form. You may use $36.87° = \tan^{-1}\left(\frac{3}{4}\right)$.

Length $= \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$



reference angle

$= \tan^{-1}\left(\frac{3}{4}\right) = 36.87°$

Standard position angle

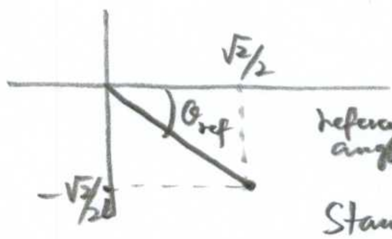
$180° - 36.87° = 143.13°$

Answer: $5 \angle 143.13°$

9. Express $\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i$ in exponential form.

polar form: length $= \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(-\frac{\sqrt{2}}{2}\right)^2} = 1$

Angle



reference angle: $\tan^{-1}\frac{\sqrt{2}/2}{\sqrt{2}/2} = \tan^{-1} 1 = 45^\circ = \frac{\pi}{4}$

Standard position angle

$360^\circ - 45^\circ = 2\pi - \frac{\pi}{4} = \frac{7}{4}\pi$.

$1 \angle \frac{7}{4}\pi \xrightarrow{\text{exponential form}} 1 \cdot e^{\frac{7}{4}\pi}$

10. Find all three roots of $z^3 = 1$, where z is a complex variable.

$1 = 1 \cdot \left(\cos(0 + 2n\pi) + i\sin(0 + 2n\pi)\right)$

cubic roots of $1 = \{ 1^{1/3}\left(\cos\frac{0}{3} + i\sin\frac{0}{3}\right) = 1,$

$1^{1/3}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -\frac{1}{2} + \frac{\sqrt{3}}{2}i$

$1^{1/3}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = -\frac{1}{2} - \frac{\sqrt{3}}{2}i \}$

$$\left\{ 1, \frac{-1 + \sqrt{3}i}{2}, \frac{-1 - \sqrt{3}i}{2} \right\}$$