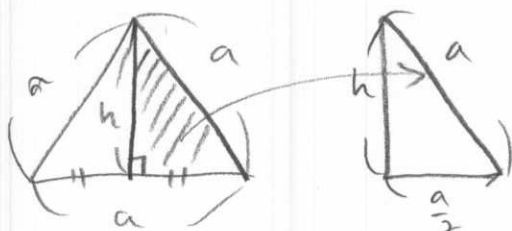


Midterm Examination 2
MTH 13 Section E01
21 March 2017 18:00 – 19:50

Instructions: Please answer the following and be sure to show your work or support your answer. You are not allowed to use the textbook, workbook, or notes. You cannot talk to other students. You can use your calculator. You also can refer a 1-page formula sheet of your own, upon approval.

1. Let T be an equilateral triangle with side length x . Express the area of T as a function in x .



By Pythagorean theorem,

$$h^2 + \left(\frac{a}{2}\right)^2 = a^2$$

$$\Leftrightarrow h^2 = a^2 - \frac{a^2}{4} = \frac{3}{4}a^2$$

$$h = \pm \sqrt{\frac{3}{4}a^2} = \frac{\sqrt{3}}{2}a$$

$$\text{Area} = \frac{1}{2}a \cdot h = \frac{1}{2}a \cdot \frac{\sqrt{3}}{2}a = \frac{\sqrt{3}}{4}a^2$$

2. Let $f(x) = x^2 + 1$. Compute

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 1 - (x^2 + 1)}{h} = \frac{x^2 + 2xh + h^2 + 1 - x^2 - 1}{h}$$

$$= \frac{2xh + h^2}{h} = \underline{\underline{2x+h}}$$

2

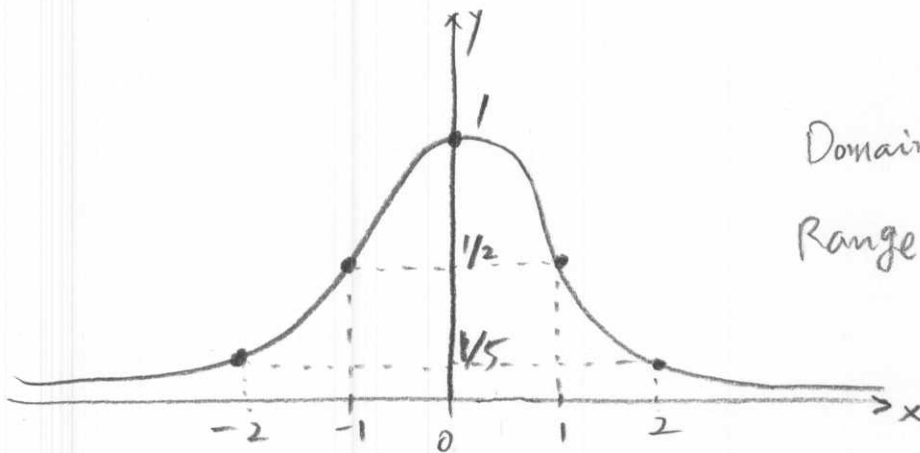
3. Draw the graph of

$$f(x) = \frac{1}{x^2 + 1}$$

satisfying the following criteria:

- $f(0) = 1$, $f(1) = 1/2$, and $f(2) = 1/5$.
- As x gets bigger, the value of $f(x)$ gets smaller, while having positive values.
- $f(x) = f(-x)$.

Also find the domain and the range of the function f .

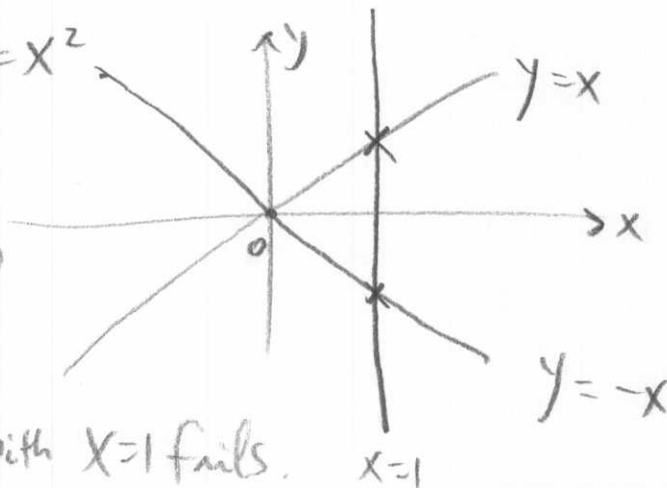


Domain: all real numbers
Range: $\{y : 0 < y \leq 1\}$

4. Draw the graph of $y^2 = x^2$ and explain that the graph is not a graph of a function.

Any (x, y) on the graph of $y = x$ or $y = -x$

Satisfy $y^2 = x^2$



This is not a graph of function because the vertical line test with $x=1$ fails.

5. Suppose you deposited \$100 to an ETF with 7.68% of APR.

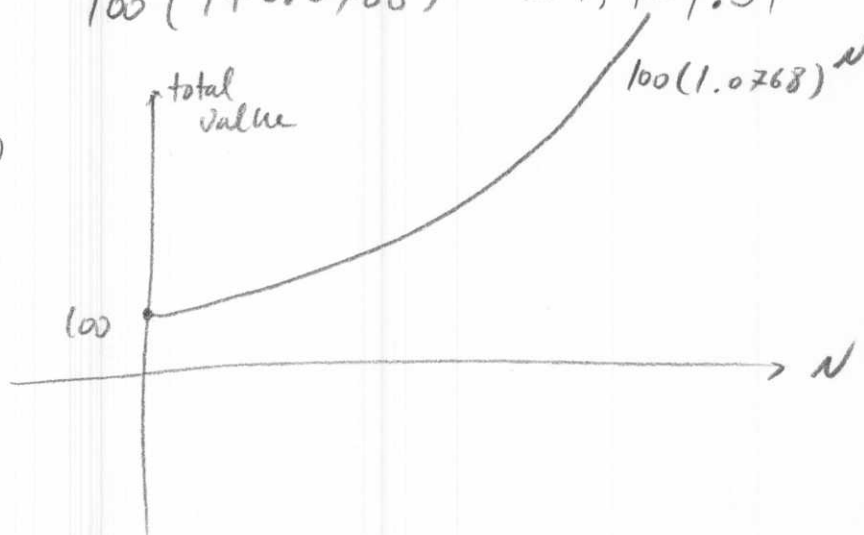
(1) What is the total value of the investment after 40 years?

(2) Plot the graph of function obtained from (1) in the xy -plane. Here x -axis stands for years (N) and the y -axis the total value of the investment when \$100 was deposited initially.

Note that the S&P 500 index recorded 20 years average APY 7.68%. Note also that $(1.0768)^{20} = 4.4$ and $(1.0768)^{40} = 19.3$.

(1) $100(1 + 0.0768)^{40} = 1,929.31$

(2)



4

6. Solve for x :

$$\log(3x+1) + \log 4 = \log(2x+1)$$

$$\text{LHS} = \log 4 \cdot (3x+1) = \text{RHS}$$

$$x > -\frac{1}{3} \quad \text{and} \quad x > -\frac{1}{2}$$

$$\Leftrightarrow 4 \cdot (3x+1) = 2x+1$$

$$12x+4 = 2x+1$$

$$10x = -3$$

$$x = -\frac{3}{10}$$

7. Solve for x : $4^{3x-4} = 2^{2x+3}$

$$(2^2)^{3x-4} = 2^{2x+3}$$

$$\Leftrightarrow 2^{6x-8} = 2^{2x+3}$$

$$\Leftrightarrow 6x-8 = 2x+3$$

↑
apply \log_2
on both sides

$$4x = 11$$

$$x = \frac{11}{4}$$

8. Simply $\log_{\frac{1}{36}} 6$ so that the final form does not have log in it.

$$\begin{aligned}\log_{\frac{1}{36}} 6 &= \log_{6^{-2}} 6 = \log_{6^{-2}} (6^{-2})^{-\frac{1}{2}} \\ &= -\frac{1}{2} \boxed{\log_{6^{-2}} 6^{-2}} = -\frac{1}{2}\end{aligned}$$

9. Use the properties of logarithm and expand the following expression into a form that only contains sum or difference of log or numbers.

$$\log_2 \frac{x^{-2}y^3}{\sqrt[3]{2}} = *$$

$$* = \log_2 x^{-2} + \log_2 y^3 - \log_2 2^{\frac{1}{3}}$$

$$= -2 \log_2 x + 3 \log_2 y - \frac{1}{3} \boxed{\log_2 2}$$

$$= -2 \log_2 x + 3 \log_2 y - \frac{1}{3}$$

10. Draw the graph of $y = (100)^x$ in the $x(\log y)$ -plane.

$$\log y = \log 100^x = x \log 100 = 2x.$$

