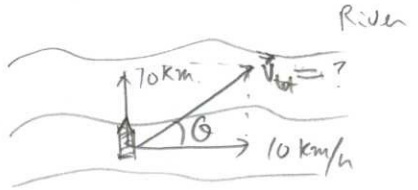
Final Examination
MTH 13 Section E01
25 May 2017 18:00 – 19:55

Instructions: Please answer the following and be sure to show your work or support your answer. You are not allowed to use the textbook, workbook, or notes. You cannot talk to other sudents. You can use your calculator. You also can refer a 1-page formula sheet of your own, upon approval.

1. A river flows at the rate of 10 km/h, A rower who can travel 10 km/h in stillwater, heads directly across the current. Find the rate and direction of travel of the boat.



$$|\vec{v}_{tot}| = \sqrt{10^2+10^2} = 10\sqrt{2} \text{ km/h}$$

$$\theta = \tan^{-1}\frac{10}{10} = \tan^{-1}(1) = \frac{\pi}{4}.$$

$$\vec{v}_{tot} = 10\sqrt{2} \angle \frac{\pi}{4}$$

2. Express the following complex number in the form of $a + bi$:

$$\frac{5+12i}{4-5i}.$$

$$\frac{5+12i}{4-5i} = \frac{(5+12i)(4+5i)}{(4-5i)(4+5i)} = \frac{20+60i^2+48i+25i}{16+25} = \frac{-40+73i}{41}$$

$$= -\frac{40}{41} + \frac{73}{41}i.$$

3. Find all three roots of $z^3 = 1$, where z is a complex variable.

$1 = 1 \angle 0^\circ$

$z_1 = 1$

$z_2 = 1 \cdot e^{\frac{2\pi}{3} i}$

$z_3 = 1 \cdot e^{\frac{4\pi}{3} i}$.

4. Let $f(x) = x^2 + 2017$. Compute

$$\frac{f(x+h) - f(x)}{h}.$$

$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 2017 - x^2 - 2017}{h} = \frac{2xh + h^2}{h} = 2x + h$.

5. Suppose \$100 is deposited with 7% of APR. What is the total value of the investement after 40 years?

Principal amount \$100.

APR 7%

40 years investment.

$$\text{total value} = 100 \cdot (1+0.07)^{40} \approx 1,497.46$$

6. Solve for x: $\log_5(x^2 - 5x + 1) = 2$.

$$\Leftrightarrow x^2 - 5x + 1 = 25$$

$$x^2 - 5x - 24 = 0.$$

$$(x-8)(x+3) = 0$$

$$x = 8 \text{ or } x = -3.$$

when $x = 8$,

$x^2 - 5x + 1 = 64 - 5 \cdot 8 + 1 = 25$

so $\log_5 25 = 2$. ✓

when $x = -3$

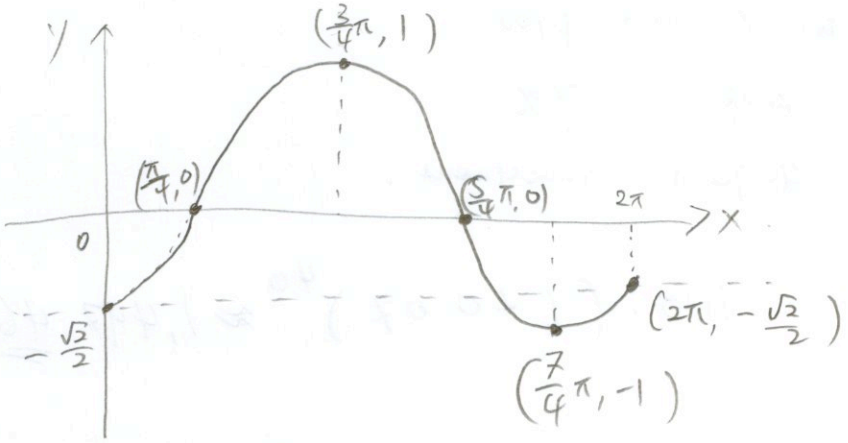
$x^2 - 5x + 1 = 9 - 5(-3) + 1$

$= 9 + 15 + 1 = 25$

so $\log_5 25 = 2$. ✓

Solutions: $\{8, -3\}$.

7. Draw the graph of $y = \sin(x - \frac{\pi}{4})$ where $0 \leq x \leq 2\pi$.



8. Prove the following identity:

$$(\tan x + \cot x) \sin x \cos x = 1.$$

$$(\tan x + \cot x)\sin x \cos x = \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right) \sin x \cos x$$

$$= \sin^2 x + \cos^2 x = 1. \quad \checkmark$$

9. Prove the following identity:

$$\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}.$$

$$RHS = \frac{1-\tan^2 x}{\sec^2 x} = \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{\frac{1}{\cos^2 x}} = \cos^2 x - \sin^2 x = \cos 2x = LHS. \checkmark$$

10. Use Cramer's rule to solve the following linear system:

$$\begin{cases} x + y + z = 1 \\ x - z = 1 \\ x + y = 1 \end{cases}$$

$$x = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} \Big/ \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} = 1$$

$$y = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} \Big/ \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

$$z = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} \Big/ \begin{vmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} = 0$$