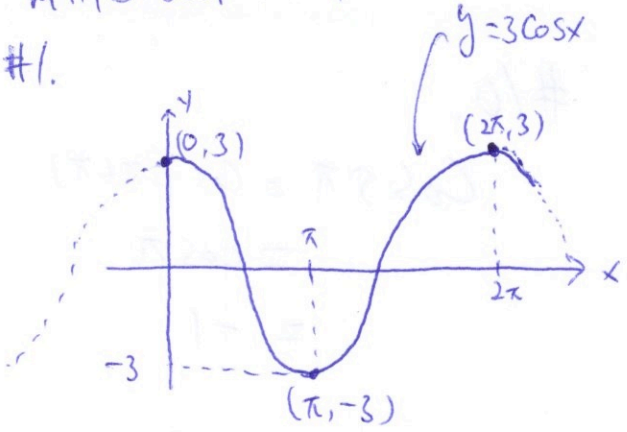
MTH13 Section E01 EXAM 3 Review Sheet Solution

#1.



#2. $y = \frac{1}{4}\cos\frac{\pi}{4}x$.

Amplitude $= 1/4$.

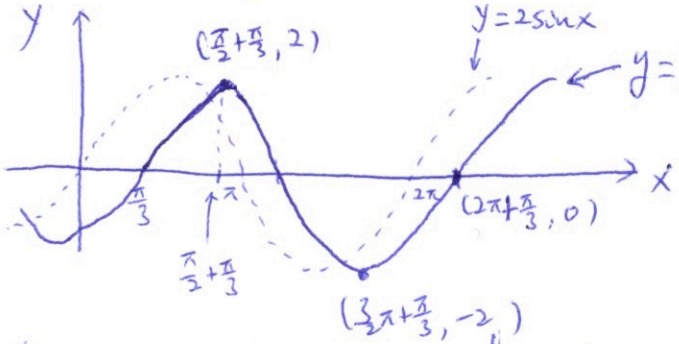
Period $= \frac{2\pi}{\frac{\pi}{4}} = \frac{2\cdot 4}{\frac{1}{4}\cdot 4} = 8$.

#3. $y = 2\sin(x - \frac{\pi}{3})$

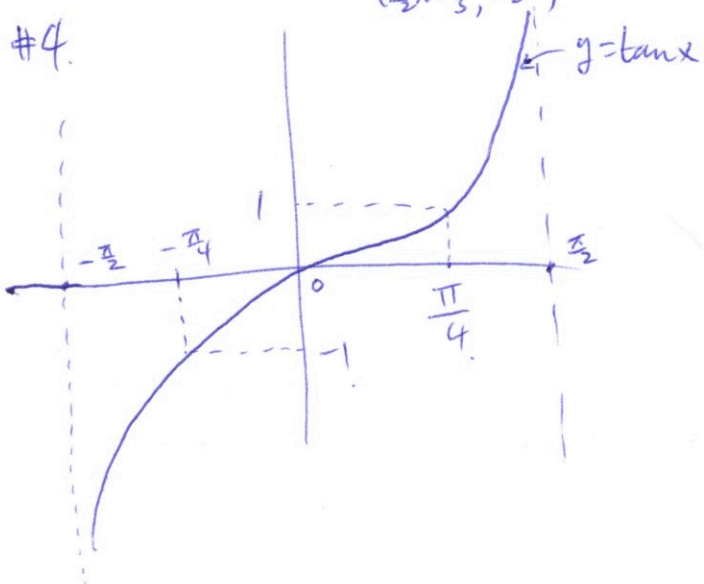
Amplitude 2

period 2π

phase-Shift $\frac{\pi}{3}$



#4.



#5 $\frac{1-\sin x}{\sin x \cot x} = \frac{\cos x}{1+\sin x}$

$$LHS = \frac{1-\sin x}{\sin x \frac{\cos x}{\sin x}} = \frac{1-\sin x}{\cos x} = \frac{1-\sin x}{\cos x}\frac{(1+\sin x)}{(1+\sin x)}$$

$$= \frac{1-\sin^2 x}{\cos x(1+\sin x)} = \frac{\cos^2 x}{\cos x(1+\sin x)} = \frac{\cos x}{1+\sin x}$$

$$= RHS \checkmark$$

#6

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

$$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4} \checkmark$$

#7 $\frac{\cos 3x}{\sin x} - \frac{\sin 3x}{\cos x}$

$$= \frac{\cos 3x}{\sin x}\frac{\cos x}{\cos x} - \frac{\sin 3x}{\cos x}\frac{\sin x}{\sin x}$$

$$= \frac{\cos 3x \cos x - \sin 3x \sin x}{\sin x \cos x}$$

$$= \frac{\cos 4x}{\sin x \cos x} = \frac{2\cos 4x}{\sin 2x}.$$

$$= \frac{2 - 4\sin^2 2x}{\sin 2x} = 2\csc 2x - 4\sin 2x.$$

#8. $\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}} = \pm\sqrt{\frac{1+\cos x}{2}\cdot\frac{1-\cos x}{1-\cos x}}$.

$$= \pm\sqrt{\frac{1-\cos^2 x}{2(1-\cos x)}} = \pm\sqrt{\frac{\sin^2 x}{2(1-\cos x)}}$$

$$= \sin x \cdot \left(\pm\sqrt{\frac{1}{4(1-\cos x)}}\right)$$

$$= \sin x \cdot \left(\frac{1}{2\sin\frac{x}{2}}\right) = \frac{\sin x}{2\sin\frac{x}{2}}.$$

#9.

$$\cos\frac{x}{2} = 1+\cos x.$$

$$\Leftrightarrow \pm\sqrt{\frac{1+\cos x}{2}} = 1+\cos x$$

$$\Leftrightarrow \frac{1+\cos x}{2} = (1+\cos x)^2$$

$$= 1+2\cos x+\cos^2 x.$$

$$\Leftrightarrow 1+\cos x = 2+4\cos x+2\cos^2 x.$$

$$\Leftrightarrow 2\cos^2 x+3\cos x+1 = 0$$

$$(2\cos x+1)(\cos x+1) = 0.$$

$$\cos x = -\frac{1}{2} \quad \text{or} \quad \cos x = -1.$$

$$x = 120^\circ, 300^\circ \qquad x = 180^\circ$$

$x = 120^\circ$ $\quad \cos 60^\circ = 1+\cos 120^\circ$

$$\frac{1}{2} = 1+\left(-\frac{1}{2}\right) \checkmark$$

$x = 300^\circ$ $\quad \cos 150^\circ = 1+\cos 300^\circ$

$$-\frac{\sqrt{3}}{2} \neq 1+\left(-\frac{1}{2}\right)$$

not a solution!!

$x = 180^\circ$ $\quad \cos\frac{180^\circ}{2} = 1+\cos 180^\circ$

$$0 = 1+(-1) \checkmark$$

Answer $x = 120^\circ, 180^\circ$

or $\frac{2\pi}{3}, \pi.$

#10.

$$\cos 5\pi = \cos(4\pi+\pi)$$

$$= \cos\pi.$$

$$= -1.$$

$$\cos^{-1}(-1) = \pi.$$

answer.