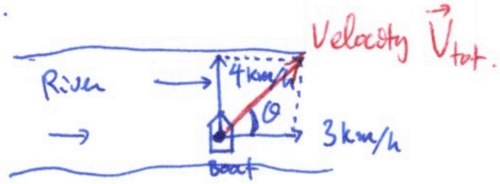
Final Exam Review. MTH13 Section E01.

#1.



$$|\vec{V}_{tot}| = \sqrt{3^2+4^2} = 5.$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^\circ$$

Answer: $\underline{5 \angle 53.13^\circ}$.

#2. $$\frac{-4-3i}{-1-2i} = \frac{\cancel{(-1)}(4+3i)}{\cancel{(-1)}(1+2i)} = \frac{(4+3i)(1-2i)}{(1+2i)(1-2i)}$$

$$= \frac{4+3i-8i-6i^2}{1^2+2^2} = \frac{10-5i}{5}.$$

$$= \underline{\underline{2-i}}.$$

#3. Note that $i = 1\angle\frac{\pi}{2}$

So three roots of $z^3=i$ are

$$z_1 = e^{i\cdot\frac{\pi/2}{3}} = e^{\frac{\pi}{6}i}.$$

$$z_2 = e^{i\cdot\frac{2\pi+\frac{\pi}{2}}{3}} = e^{\frac{5}{6}\pi i}.$$

$$z_3 = e^{i\cdot\frac{4\pi+\frac{\pi}{2}}{3}} = e^{\frac{3}{2}\pi i}.$$

#4. $$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2+(x+h)+2017-(x^2+x+2017)}{h}$$

$$= \frac{\cancel{x^2}+2xh+h^2+\cancel{x}+h+\cancel{2017}-\cancel{x^2}-\cancel{x}-\cancel{2017}}{h}$$

$$= \underline{\underline{2x+1+h}}.$$

#5.

Principal \$200. APR 3%. 10 years.

$$\text{Total Value} = 200(1+0.03)^{10} \approx 361.22.$$

#6. $$\log_3(6x^2-5x+23) = 3.$$

$$\Leftrightarrow 6x^2-5x+23 = 3^3 = 27.$$

$$\Leftrightarrow 6x^2-5x-4 = 0.$$

$$\Leftrightarrow (2x+1)(3x-4) = 0$$

$$\Leftrightarrow x = -\tfrac{1}{2} \text{ or } x = \tfrac{4}{3}.$$

when $x=-\frac{1}{2}$, $6x^2-5x+23 > 0$ ✓

$$= \frac{6}{4}+\frac{5}{2}+23 = \frac{16}{4}+23 = 27.$$

So $\log_3 27 = 3$ ✓

when $x=\frac{4}{3}$ $6x^2-5x+23 > 0$ ✓

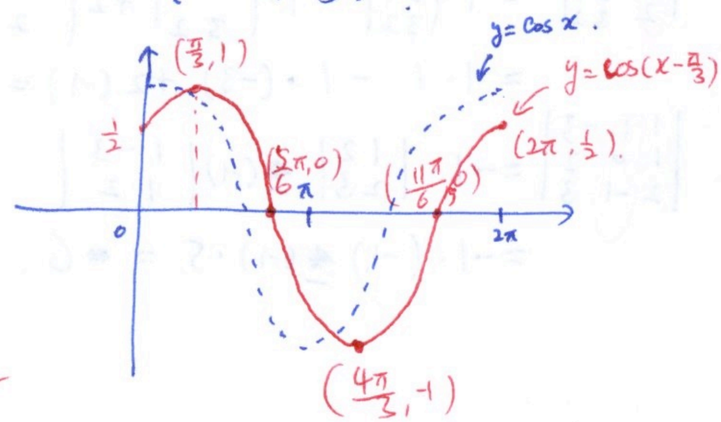
$$6\cdot\frac{16}{9}-\frac{20}{3}+23 = \frac{96-60}{3^2}+23$$

$$= \frac{36}{9}+23 = 4+23 = 27.$$

So $\log_3 27 = 3$ ✓.

Solutions $= \left\{-\frac{1}{2}, \frac{4}{3}\right\}$.

#7.



#8. $\sec x + \tan x + \cot x$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{\sin x + \sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$= \frac{\sin x + 1}{\sin x \cos x} \quad ✓$$

#9. $$\frac{\sin(x-y)}{\sin(x+y)} = \frac{\sin x\cos y - \cos x \sin y}{\sin x \cos y + \cos x \sin y} \cdot \frac{\frac{1}{\cos x\cos y}}{\frac{1}{\cos x \cos y}}$$

$$= \frac{\frac{\sin x}{\cos x} - \frac{\sin y}{\cos y}}{\frac{\sin x}{\cos x} + \frac{\sin y}{\cos y}} = \frac{\tan x - \tan y}{\tan x + \tan y}. \quad ✓$$

#10. By Cramer's rule,

$$x = \begin{vmatrix} -3 & 1 & -1 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \end{vmatrix} \Big/ \begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & -1 & 2 \end{vmatrix} = -2/2 = -1$$

$$y = \begin{vmatrix} 1 & -3 & -1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{vmatrix} \Big/ \begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & -1 & 2 \end{vmatrix} = 2/2 = 1$$

$$z = \begin{vmatrix} 1 & 1 & -3 \\ 1 & 0 & 2 \\ 2 & -1 & 3 \end{vmatrix} \Big/ \begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & -1 & 2 \end{vmatrix} = 6/2 = 3$$

$$\begin{vmatrix} 1 & 1 & -1 \\ 1 & 0 & 1 \\ 2 & -1 & 2 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} + (-1) \cdot \begin{vmatrix} 1 & 0 \\ 2 & -1 \end{vmatrix}$$

$$= 1 \cdot 1 - 1 \cdot 0 + (-1) \cdot (-1) = 2.$$

$$\begin{vmatrix} -3 & 1 & -1 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \end{vmatrix} = -1 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} - (-1) \begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= -1 - (-1) \cdot (-1) = -2.$$

$$\begin{vmatrix} 1 & -3 & -1 \\ 1 & 2 & 1 \\ 2 & 3 & 2 \end{vmatrix} = 1 \cdot \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} - 1 \cdot \begin{vmatrix} -3 & -1 \\ 3 & 2 \end{vmatrix} + 2 \begin{vmatrix} -3 & -1 \\ 2 & 1 \end{vmatrix}$$

$$= 1 \cdot 1 - 1 \cdot (-3) + 2(-1) = 2.$$

$$\begin{vmatrix} 1 & 1 & -3 \\ 1 & 0 & 2 \\ 2 & -1 & 3 \end{vmatrix} = -1 \cdot \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} - (-1) \begin{vmatrix} 1 & -3 \\ 1 & 2 \end{vmatrix}$$

$$= -1 \cdot (-1) - (-1) \cdot 5 = -6.$$