

A solution to **Problem 8.29** (Problem 8.33 in Korean edition). Let  $\gamma(u) = (\cos u, \sin u, 0)$  and  $\mathbf{e}_3 = (0, 0, 1)$ .

$$\mathbf{x}^+(u, v) := \gamma(u) + v\gamma'(u) + v\mathbf{e}_3$$

$$\mathbf{x}^-(u, v) := \gamma(u) - v\gamma'(u) + v\mathbf{e}_3$$

If one writes explicitly,  $\mathbf{x}^+(u, v) = ((\cos u - v \sin u), (\sin u + v \cos u), v)$ . Hence  $(\cos u - v \sin u)^2 + (\sin u + v \cos u)^2 - v^2 = 1 + v^2 - v^2 = 1$ . One can similarly verify for the case of  $\mathbf{x}^-(u, v)$ .

**Click here** for a very nice video on how a hyperboloid is constructed as a ruled surface and it is applied to the construction of the Sagrada Familia Cathedral.