Complex Analysis I
Course Outline
Course 7412010 Section 01, Spring 2021
Tuesdays 13:00 - 13:50, Thursdays 10:00 - 11:50, Room: E1-2 #306
Chungbuk National University

Instructor: Dr. Byungdo Park

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Office hours: By appointment.

Class webpage: Announcements, homework, exam schedules and other relevant information will be posted on the following webpage: https://byungdo.github.io/teaching/s2021_cv1.html which is also accessible via instructor's webpage: https://byungdo.github.io/

Textbook:

- Joseph Bak and Donald Newman, *Complex Analysis*, 3rd Edition (2010), Springer, ISBN-13: 9781441972873.

References:

- Lars V. Ahlfors, *Complex Analysis*, 3rd Edition (1979), McGraw Hill Higher Education, ISBN-13: 9780070850088.
- Elias M. Stein and Rami Shakarchi, *Complex Analysis (Princeton Lectures in Analysis, No. 2)*, 1st Edition (2003), Princeton University Press, ISBN-13: 9780691113852.
- Otto Forster, *Riemannsche Flächen (Heidelberger Taschenbücher, Band 184)* 1. Edition (1977), Springer Berlin Heidelberg, ISBN-13: 9783540080343
- Bernard Maskit, *Kleinian Groups (Grundlehren der mathematischen Wissenschaften 287)* 1st Edition (1988), Springer Berlin Heidelberg, ISBN-13: 9783540177463
- Saeed Zakeri, *A Course in Complex Analysis*, to be published by Princeton University Press.

Prerequisites: A solid coursework on calculus. Analysis I and II (7412029, 7412022) are recommended and results therein will not be repeated in this course but not strictly required. The instructor does not dissuade students without meeting the prerequisite criteria registering for this course at his/her own risk.

Course description: This is the first-semester course of a year-long course, "Complex Analysis" which consists of two parts, Complex Analysis I (7412010) and Complex analysis II (7412007). We will study basic analysis of complex valued functions. We will learn that holomorphicity and complex-analyticity coincide, which is an amazing fact. This fact leads us to some of the most important tools in complex analysis. One being, Cauchy-Goursat theorem and another being Cauchy's

integral formula. We will touch on other great theorems in complex analysis such as Liouville's theorem, Morera's theorem, the open mapping theorem, and Schwarz lemma. Then we will dive into conformal mappings and Möbius transformations to prepare for our great finale, the Riemann mapping theorem.

Complex analysis is a fantastic subject in mathematics that one can pioneer through all his life. It is also arguably the core of modern mathematics research. To name a few examples; measuring volumes of moduli spaces of Riemann surfaces with marked points (Maryam Mirzakani), holomorphic dynamics (John Milnor), hyperbolic 3-manifolds, and geometrization theorem (Bill Thurston). Therefore, I would encourage you all to take this course with me so I can share with you this wondrous subject.

There are two types of people in this world, ones who choose to remain in Plato's cave and those whose curiosity leads them astray to the outside world. It is the duty of every teacher to guide those who are curious. The primary focus of the instructor for this course is to provide a uniform and broad understanding of complex variables at the undergraduate level. This course will include/cover the subtopics which will show up on the national examination for secondary school teachers but it will also cover subtopics that will not necessarily be on the national exam. Naturally, this means the students will be asked questions on in-class exams that will not show up on the national exam. I would like to make the point that one should take this course with excitement. If you feel that this course is too overwhelming because it includes more than what is on the national exam I would suggest you take a different course or reconsider taking this course. The goal of this course is to go beyond just the national exam and truly enjoy the beauty of mathematics. I would like to share this experience with all of you, so ask yourself, what kind of person do you want to be?

Course objectives: At the end of the course students should be able to:

- Understand holomorphicity and complex analycity coincide.
- List conditions equivalent to holomorphicity.
- Evaluate complex integrals.
- Understand the Cauchy-Goursat theorem and the Cauchy integral formula.
- Appreciate the difference of real and complex analysis via theorems on entire functions.
- Know how to use the open mapping theorem and Schwarz' lemma.
- Observe the homotopy theory interplaying with Cauchy's theorem.
- Understand Möbius transformations and their importance in geometry.
- Precisely state what the Riemann mapping theorem is and what the theorem means.
- Create an online learning contents such as YouTube videos for sharing knowledge with a broader audience.
- Discuss various implications on civic education from mathematics we appreciate in this course.

- Shape an overarching perspective on teching complex numbers and the complex plane in secondary school mathematics curricular.

Aspects on civic education: This course has been redesigned to incorporate strategies and goals of **The Project for Strengthening Civic Educational Competence in Teacher Training Colleges** (PI: Prof. Dr. Jongyeun Lee in the Department of Education) of the College of Education, Chungbuk National University. While the appreciation of complex analysis will be tightly intertwined with civic educational themes, we shall follow the following specific weekly plans.

- **Week 1: The fairness.** The administrative introduction to this course is a great source to discuss values we strive in civic education. Using a carefully designed syllabus we shall look into details of the agreement between the instructor and students to see how everything coalesces to a single concept, the fairness. In addition, we shall add an aspect of character education on professional communication in electronic writing, such as how to compose an email to professionals for professional affairs. Relevant items in Harmony-CHAMP abilities listing: C1, C2, C3, H1, H2, H3, P2, P3.

- **Week 5: The correctness.** It is a well-known that the original proof of Cauchy's integral theorem had a gap assuming that the derivative of the holomorphic function is continuous. It took near 75 years for E. Goursat removing the condition. As such mathematicians pursuing an indestructible and flawless theories does not necessarily mean that mathematics is consisting of undoubtable facts. It is more like *La Sagrada Familia* mathematicians over generations try to build a more distilled and flawless logical systems. Welcoming mistakes or incomplete ideas is more desirable and purposeful in mathematics than tossing them out. In this week, while looking into details of Cauchy's and Goursat's proofs, we shall discuss the importance of welcoming imperfect form of arguments or discourse and expand the discussion how to strengthen our communication skills and conflict-managing skills in classroom. The goal is not to become an educator who mindlessly repudiates students with mistakes. You will learn how to find a flickering lights buried in students' mistakes and errors, and as a result you will never think of stupid questions as stupid questions. Relevant items in Harmony-CHAMP abilities listing: C1, C2, C3, H1, H2, H3, A1, A2, A3, M1, M2, M3, P1, P2, P3.

- **Week 14: The interpretation.** The Riemann mapping theorem (RMT) states that every nonempty simply connected open sets in the complex plane is biholomorphic to the open unit disc under a unique biholomorphic map. However this theorem never constructs the map. We can say only the existence and the uniqueness without knowing the map. It is known that the constructive RMT requires additional restrictions on the domain. Apparently, RMT is a mathematical truth only at its face value. The majority of social conflicts arise due to confusions on terms and wordings in statements of a discourse, or more frequently, laws and policies lacking clarity cause confusions and conflicts. The Supreme Court gave out several precedents how one can interpret laws but it could mean two totally different guidelines on a specific matter of law interpretations we face (See 대법원 2009. 4. 23. 선고 2006다81035 판결, 판시요지 [1] 법률 해석의 방법과 한계). This week, we shall look into statement of the RMT and expand our discourse in aforementioned direction to enrich our understandings on logical communications in civic education. Relevant items in Harmony-CHAMP abilities listing: C1, C2, C3, H1, H2, H3, A1, A2, A3, M1, M2, M3, P1, P2, P3.

On Pilot Operation of the Future Education Center: As we have seen in the year 2020 pandemic situation due to SARS-CoV-2 outbreak, there have been increasing needs of teachers' abilities in remote, non-facing, and online platforms carrying out the entire school education in an equal competence. The government has chosen 10 teacher training institutions including us to set up facilities for strengthening such abilities of future teachers. This course is participating in the **Chungbuk National University Future Education Center Construction and Pilot Operation Project** (PI: Prof. Dr. Dongseok Yang in the Department of Physics Education) and will carry out Project-Based Learning (PBL) in accordance with the purpose and goal of the project. More details are in "Assessment of Project-Based Learning (PBL)" below.

Details on problem solving: Problems arising in this course will be requiring proofs and calculations based on the mathematical discourse in class. Through dialogues and discussions during each lecture as well as the instructor's office hours, the instructor will guide students approaching to problems that they will have to address.

Details on class proceeding: The instructor will give lectures on the material following the weekly lesson plan and assign weekly homework problems. He will also require you to participate in a Project-Based Learning to strengthen your competence as a teacher also in online, remote setup.

Grading policies: 60% from final exam, 17% from homework, 15% from PBL project, and 8% from attendance. Absolute evaluation [A: 100–90 points, B: 89.99–80 points, C: 79.99–70 points, D: 69.99–60 points, F: less than 60 points] with curving. Here the curving means a horizontal shift of the bell-shaped curve of %-score distribution in either directions using a rational constant which is determined at the discretion of the instructor. Grading policies in the academic integrity policies are applied in higher priority than the above grading policies.

Homework policies: A list of homework problems will be posted on the class webpage roughly in weekly basis. Late homework will be accepted. The instructor will assign as many homework problems as it is needed to master the subject. The instructor will scan through each submitted homework and assign a score 2, 1, or 0 depending on quality of work. The homework score for the total grade will be calculated based on the following formula: $(\sum_{i=1}^{h} h_i \cdot n_i)/(\sum_{i=1}^{h} 2 \cdot n_i)$, where h is total number of homework assignment, h_i is the score for the i^{th} homework score, n_i is the number of problems in the i^{th} homework.

Attendance policies: Attendance data will be collected in every class meeting and will be used for determining your final grade. You will get a grade F if you have missed more than 25% of class meeting hours. Up to 3 hour of absence there is no penalty on your score. After that, you lose 1% of total score for an absence to each 50-minute long class meeting, with a maximum total loss 8% from your total score. If you have permissible reasons for your absence in accordance with the Regulation on Academic Management of the CBNU Article 52(1) (충북대학교 학사운영규정 제52조(공결승인) 제1항), you will need to contact the Department Assistant to follow the procedure for getting an approval on your absence bringing proper documentation as proof. That said, you have to fill out a form and submit it along with appropriate proofs before the absence or after seven days of the date of absence.

Assessment of Project-Based Learning (PBL): You must record a 20-minute long video lecture about one of the following:

- A sample lecture on any topic listed on the syllabus of this course.
- A sample lecture on a concept from secondary school geometry curricular.

You should submit the video in a form of a YouTube video link by choosing the sharing option "unlisted(일부공개)." Your video will be disclosed to your classmates in this course as a part of a YouTube playlist. Registering to this course would mean that you accept sharing your video lecture with your classmates via YouTube. You may turn your video into "private" or even delete the video after your letter grade for this course is assigned. The assessment will be done as follows: 3/3 all in all good work. 2/3 lacking important examples, theorem, proofs or there are significant mathematical errors. 1/3 overall poor contents of the material. 0/3 no hand-in.

Assessment of learning: The assessment will be primarily done by the abovementioned grading policy. Nonetheless, the instructor will also take into account students' devotions and efforts for this course as well as their enthusiasm as a future educator so that those qualitative elements are not going to be neglected.

Weekly lesson plan:

Week 1: Complex numbers, the complex plane, cubic equations. [Sections 1.1–1.3] – *Online lectures using recorded videos*

Week 2: Topological aspects of complex plane. [Sections 1.4, 1.5, 2.1] – *Online lectures using recorded videos*

Week 3: Power series [Sections 2.2, 2.3] – *Online lectures using recorded videos*

Week 4: Analytic functions [Sections 3.1, 3.2] – *Online lectures using recorded videos*

Week 5: Line integrals and entire functions [Sections 4.1, 4.2] – *Online lectures using recorded videos*

Week 6: Cauchy's integral formula and entire functions [Sections 5.1, 5.2] – *Online lectures using recorded videos*

Week 7: Newton's method, fixed point iterations, the power series representation for analytic functions [Sections 5.3, 6.1] – *Online lectures using recorded videos*

Week 8: Leeway, PBL presentations – *Online lectures using recorded videos*

Week 9: Properties of analytic functions [Sections 6.2, 6.3] – *Online lectures using recorded videos*

Week 10: The open mapping theorem, Schwarz' lemma and further properties of analytic functions [Sections 7.1, 7.2] – *Online lectures using recorded videos*

Week 11: Simply connected domains [Sections 8.1, 8.2]– *Online lectures using recorded videos*

Week 12: Conformal mappings and conformal equivalences [Sections 13.1, 13.2] – *Online lectures using recorded videos*

Week 13: Special mappings, Schwarz-Christoffel transformations [Sections 13.2, 13.3] – *Online lectures using recorded videos*

Week 14: Conformal mappings and hydrodynamics, leeway. [Sections 14.1] – *Online lectures using recorded videos*

Week 15: PBL presentations, in-class final exam. – *Online lectures using recorded videos*

Accommodating disabilities in learning and assessment: The instructor is committed to providing access to all students. If you need accommodation in classroom or in assessment, you are encouraged to set up an appointment with the instructor at your soonest availability so that we can figure out the best way to acoommodate you. Possible accommodations include, but not limited to, provision of materials from lectures, permission to hire an assistant for taking notes, audio-recording lectures, and aid/assistant devices, extension of due dates for assignments, alternative assessment for in-class presentations, extension of exam hours, and provision of an accommodating exam locations and exam sheets.

Academic integrity: It is expected that you will complete all exams without giving or receiving help from anyone. Electronic devices are not allowed in any in-class exam. You may talk to other students about the homework but you must then complete the homework yourself. The grader will trust students and will not apply any prejudice. However, if the grader has found an evidence that you have violated those policies, the grader reserves the right to investigate by summoning you to come in to his office, reproduce and explain your own solutions in front of the chalkboard. If you cannot provide a coherent and consistent explanation to your own solution to a problem or do not show up to the investigation without a documented official cause and/or an emergency, the minimum punishment would be score zero to that problem and lowering your letter grades by 2 letters. (For example, if you were to receive A+, it will become C+.) In addition to that, your other homework solutions may possibly be a subject of investigation. The investigation session will be both video and audio recorded, and the result of the investigation (including video/audio recording of the investigation) can be reported to the department or the university center. You MUST drop this course if you cannot comply with this policy. There is a separate set of policies on the google form for attendance collection which you also have to comply.

Disclaimer: (1) All emails addressed to the instructor should have a title containing the course title, name, and a brief summary as well as a body starting with "Dear Professor Last name" and ending with "Sincerely, Your full name", which contains greetings, your name and department, a brief and clear purpose written politely. Any email deviating from this format will not be accepted and will be dismissed without any rejection reply. The corresponding disadvantages are solely and entirely on the student.

(2) Lectures in this course will be given in Korean, but most of written materials will be in English. For example, the course syllabus, most of boardwork, exam problems, homework, solutions to exams, course webpage, announcements, but not limited to those. English sentences to be used in this course should be understandable enough based on the regular Korean public high school curriculum. Nonetheless if your English skill is not competent enough to follow this course or understanding announcements, it is your responsibility to ask the instructor to also provide an explanation in Korean. The instructor will take those questions under an attitude of helping students' understanding, but taking into account the contents of each question, he may reject the question or advise the questioner to visit him during his office hour to ask the question about Korean translation.

(3) This course has a deadline for attendance claims and homework assignments. Any attendance claim and assignments that are past due will not be accepted.

- Attendance: Every Saturday 23:59 of the week that the video belongs.
- Assignments: Follows the deadline posted on the course webpage for each assignment. (Only email submission will be accepted, and no hard copies will be accepted.)

사전고지: (1) 담당교수에게 보내지는 모든 이메일의 제목에는 과목명, 신원, 요지가 포함되어 있어야 하며, 본문은 반드시 "OOO교수님께"로 시작하여 인사, 신원, 용건을 간단 명료하고 예의바르게 기술한 후 "OOO올림" 또는 "OOO드림"으로 끝나야 합니다. 이 형식에 어긋난 이메일은 접수하지 않으며, 반려회신 없이 종결합니다. 이에 따른 불이익은 전적으로 학생의 단독 책임입니다.

(2) 본 강좌에서 강의는 한국어로 이루어집니다만, 글의 경우 대부분 영어가 사용될 것입니다. 수업계획서, 칠판 판서의 대부분, 시험문제, 숙제, 시험문제에 대한 풀이, 강좌의 웹페이지, 공지사항 등이 예가 될 수 있으며, 이상 열거한 것들로 한정되지 않습니다. 본 강좌에서 사용될 영어 문장들은 한국의 공립 고등학교 정규 교과과정을 기초로 할 때 충분히 이해될 수 있어야 합니다만, 만약 수강생 본인의 영어실력이 본 강좌를 따라오거나 공지사항을 이해하기에 충분치 못하다면, 담당교수에게 한국어로 추가 설명을 요청하는 것은 학생 본인의 몫입니다. 담당 교수는 학생들의 이해를 도우려는 자세로 질문을 받을 것이지만, 질문의 내용에 따라 답을 하지 아니할 수도 있고, 면담시간에 개별 방문하여 질문하도록 안내할 수도 있습니다.

(3) 본 온라인강좌는 출석근거 및 과제물의 제출시한이 있습니다. 시한이 지난 출석인정신청 및 과제물은 접수하지 않습니다.

- 출석: 해당차시가 속한 주의 토요일 23:59.
- 과제물: 과제물별로 강좌 웹페이지에 게시된 제출시한 (이메일로만 과제물을 접수하며 학과 사무실 메일박스투입 등 불인정.)

General plans and outlook concerning the new Corona virus (SARS-CoV-2) outbreak: Following the guideline of the university center "학사지원과-266(2021.01.11.) 코로나19 대응 2021 학년도 제1학기 수업 운영 계획 안내" as well as the likelihood of COVID-19 infection persisting throughout the period of this course, the instructor has planned this course to run as an entirely remote, online course. Following future guidelines from the university center, this course may be turned into an off-line, classroom lectures in which case the instructor may update the syllabus

accordingly. Details of decisions made will be updated below in this document.

Remote classes using Youtube videos: We shall have remote classes using **video-recorded lectures** posted on Youtube. The **platform** will be CBNU e-Campus (Blackboard) wherein you will be able to find Youtube video links. By the class meeting time of each day, you will be provided video recordings of lectures for that day's class meeting. Your attendance will be collected by using the online system implemented on e-Campus, while you will be provided a google form (Link) to submit in case the e-Campus system does not recognize your watching activities correctly. The instructor recommends watching youtube videos while logged into your own google account so that youtube can record your history in your account.

How to hand-in your homework in remote class setup? You can hand-in your quiz via email to `byungdo@g.cbnu.ac.kr` by scanning it or using smart phone scanner apps. You have to hand-in your homework by the due date. A late submission will not be accepted for any reasons.