Theorem: Let $g : \mathbb{R}^3 \to \mathbb{R}$ a class C^m function.

Also let $c \in \mathbb{R}$ a constant. The subset $M \subset \mathbb{R}^3$

defined by

$$M = \{ (x,y,z) \in \mathbb{R}^3 : g(x,y,z) = c \} \text{ is a surface}$$

if one of $\frac{\partial g}{\partial x}$, $\frac{\partial g}{\partial y}$, or $\frac{\partial g}{\partial z}$ is nonvanishing

at any point of M.

Proof: Let $\underset{=(p_1,p_2,p_3)}{p \in M}$ and WOLG $\frac{\partial g}{\partial z}(p) = 0$.

By Implicit Function Theorem, $\exists\, h : D \subset \mathbb{R}^2 \to \mathbb{R}$

of class C^m and $(p_1, p_2) \in D$ s.t.

① $\forall_{(u,v)\in D}$, $g(u, v, h(u,v)) = c$.

② points of the form $(u, v, h(u,v))$ with $(u,v) \in D$ fill a neighborhood p of M.

Hence there is a Monge patch

$$\begin{aligned} X : D \in \mathbb{R}^2 &\longrightarrow \mathbb{R}^3 \\ (u,v) &\longmapsto (u, v, h(u,v)) \end{aligned}$$

s.t. $X(D)$ being a neighborhood of p. We're done because p: any. $\square$